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THE NEW
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BY

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PREFACE.

This volume contains a thorough treatment, practical and theoretical, of the elements of Geometrical Optics.

Great importance has been attached to practical work, and the book contains a detailed course of easy and instructive experiments, which can mostly be performed by the student himself, and which require only simple and inexpensive apparatus. Many practical illustrations are of course derived from the common phenomena of every-day life.

A special feature of the book is the large number of diagrams, for in this subject good diagrams are essential. No student can hope to master the subject of light until he has acquired the faculty of drawing clear and correct pictures tracing the course of the pencils of light which produce any image from their origin to the eye. This faculty can only be acquired by careful and constant practice.

Other points which have received a more thorough treatment than is usual in elementary text-books are dispersion, colour, vision, and aids to vision.

The book is supplied with a good collection of problems: for calculations are another very necessary element in this subject. Special care has been taken in explaining the convention of signs in optical formulae, and in training the student not to give numerical answers "correct" to an impossible number of decimal places.

Candidates for London University Matriculation who wish to follow the minimum course of reading may omit the following portions of this book:—

§§ 33, 34; all mention of convex mirrors in Chapter V.; §§ 41 (*c*, *d*), 43-45, 62, 63, 69; all mention of concave lenses in Chapters VII., VIII.; §§ 79 (2, 3), 81-84, 87, 88, 98, 101, 105, 107-117, 120-122, 128, 131-147.

This book is based upon the latest edition of the *Text-Book of Light* by Dr. R. W. Stewart, and has been recast in its present form by Professor John Satterly.

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LIGHT.

CHAPTER I.

INTRODUCTORY.

1. **Light** is the external physical cause of the sensation called sight.

2. **The Cause of Light.** Light is generally believed to have its origin in vibration of the luminous body, just as sound originates in vibration of the sonorous body, and to be transmitted to the eye as sound is to the ear by means of undulations in the intervening medium. There are, however, these important differences between the two phenomena. First, that whereas the sonorous body vibrates as a whole, it is only the *molecules* of the luminous body that are in vibration. Secondly, that while the frequency of sonorous vibrations is only a few hundreds or thousands per second, the frequency of luminous vibrations amounts to *hundreds of billions* (about 400 to 800 billions) per second.

Bodies which of themselves produce light are said to be *self-luminous.* Many bodies are able to reflect light from self-luminous objects, and thus become luminous themselves. Thus the sun is self-luminous, but the moon is rendered luminous only by reflecting the sun's light.

3. **The Transmission of Light. The Luminiferous Ether.** A further point of difference between light and sound is found in the medium by which they are propagated. A

properly isolated bell sounding in a good vacuum is quite inaudible, but it is no less visible than before removal of the air. And we know that light from the stars reaches our eyes after traversing millions of millions of miles of "empty" space. As our senses give us no evidence of anything in these vacuum spaces, physicists have been forced to fill them with an *imaginary substance* which they call the *luminiferous ether.** It is quite unlike any form of matter with which we are acquainted. It is probably devoid of weight, and is perfectly elastic, and so far would appear to most nearly resemble an exceedingly rarefied gas; but the undulations it transmits are known to be transverse, and such therefore as no gas, by reason of its being devoid of cohesion, is competent to transmit. The kind of matter to which it appears to bear the closest resemblance is an extremely attenuated jelly.

This ether is supposed not only to occupy all space, but to interpenetrate all matter, and to lie between the molecules of even the densest solids, as air lies between the leaves and branches of a tree.

It may now be possible for us to picture to ourselves the vibration of the molecules of the luminous body setting up undulations in the ether which travel with inconceivable speed† in all directions. Some of these undulations falling on the eyes, there set up changes in the optic nerve, which, when transmitted to the brain, produce the sensation of light. But whatever may be the nature of light, there are some fundamental properties, established by experiment, which may be studied quite independently of any hypothesis on this point.

In the pages that follow we propose to study in this way a few of the more important of these fundamental properties; but, as the undulatory theory is now so completely established, reference will be made to it whenever it seems advisable.

---

* This must not be confounded with the very tangible liquid called ether by chemists.

† About 186,000 miles per second, a speed that would carry it about 7½ times round the earth in a second.

4. **Light is invisible.** Remembering what light is—simply undulations in an invisible medium—this statement ought to cause little surprise. When we apparently see a beam of sunlight entering through a small window into an otherwise dark room, what we really see is not the light itself, but a number of floating particles in the air illuminated by the beam. Many of these are so large as to be easily visible separately as dancing motes. If a lighted Bunsen burner be brought below the beam so as to burn up or volatilise these particles, the luminous track will be interrupted by what appears to be black smoke rising from the flames. But the Bunsen flame is perfectly smokeless, and the black spaces are full of dust-free air, consequently there is nothing in those parts to reflect the light, and it remains invisible.

5. A **Medium** is the name given to any substance through which light passes. A *transparent medium* is one which transmits the light which enters it more or less completely. Possibly no medium except the ether allows all the light that enters it to pass through; a portion of the light is reflected or absorbed by the medium, and that which emerges is consequently less bright. However, a medium which transmits the greater portion of the light entering it is called *transparent*, *e.g.* water, glass, mica. Media which permit little or none of the light which falls on them to pass through are *opaque*, *e.g.* wood, iron, lampblack. Media which transmit light to some extent but do not enable one to see clearly through them are called *translucent*, *e.g.* wax, ground glass, china.

Bodies which in their ordinary state appear perfectly opaque, really transmit a very considerable amount of light when obtained in sufficiently thin laminae.

If a gold-leaf, supported between two pieces of glass, be held to the light, it will appear semi-transparent and bright green. A stone may be ground down sufficiently thin to become transparent. A piece of cardboard, which under ordinary conditions appears perfectly opaque, transmits much light when held close before an electric arc; and if the hand be similarly held it is possible to see the bones through the semi-transparent flesh. On the other hand, water, though very transparent, absorbs so much light as to make the sea-bottom below a few hundred fathoms perfectly dark Therefore the terms *transparent*, *translucent*, and *opaque* refer to a difference in degree more than in kind, and for this reason it is perhaps more correct to speak of transparent, translucent, and opaque *bodies* than to apply these terms to *substances*.

A medium is *homogeneous* when it is uniform throughout in composition, structure, and properties. A medium which is not uniform is called *heterogeneous*.

6. **Ray—Beam—Pencil.** These terms are of such frequent use in connection with light that it becomes necessary to define them. A *ray* is strictly only a mathematical conception, a line which may be taken as lying in a direction at right angles to the wave fronts.*

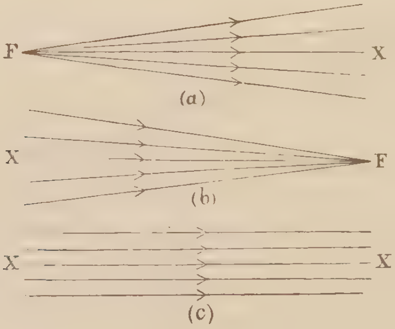

Fig. 1.

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* In other media than air, glass, and liquids, the rays are not necessarily at right angles to the wave fronts. The direction of the ray at a point may be defined as the straight line joining the centre of a small spherical obstacle, situated at that point, to the centre of the shadow produced by it on a screen, an infinitely small distance beyond it in the direction in which the light is travelling.

A *beam* of light is a collection of adjacent rays, and may be *divergent*, *convergent*, or *parallel*—that is, the component rays may diverge from, or converge to, a point, or run parallel (Fig. 1, *a*, *b*, *c*). A convergent or divergent beam has the form of a cone of small, but finite angle, while a parallel beam is a cylinder of small cross section. The *axis* of a beam, F X (Fig. 1), is the central ray passing along the geometrical axis of the figure of the beam, and the point, F, from or to which the rays of a beam diverge or converge is called its *focus*. The focus of a parallel beam is at infinity. *Pencils* are very narrow beams, and like beams may be parallel, divergent, or convergent. When light comes from a point on a very distant source such as the sun, moon, or a star, it is considered to be parallel, although, strictly speaking, it is very slightly divergent.

CHAPTER II.

RECTILINEAR PROPAGATION OF LIGHT.

7. Light travels in straight lines through the same homogeneous medium. Many familiar phenomena point to the fact that light travels through the same homogeneous medium in straight lines. If two screens be each pierced with a small hole and then held one in front of the other, in such a position (Fig. 2) that the two holes and a candle flame are in the same straight line, a ray of light can pass from the candle through the holes to an eye placed in the same straight line behind the screens; but, if either of the screens be but slightly displaced in its own plane, the candle becomes invisible. Similarly, if a scale be laid on the bottom of a vessel, and looked at over the edge of the vessel, in the way indicated in Fig. 3, it will be found that the line E A S is a straight line. In both these experiments the same medium (air) extends between the eye and the object seen; but if, in the latter example, water be poured into the vessel, it will be found that a point, S′, on the scale can be seen, and that E A O S′ is not a straight line. Hence, when light passes from one medium to another, it is in general bent out of its direct rectilinear path; and, from what has been said, it is evident that the

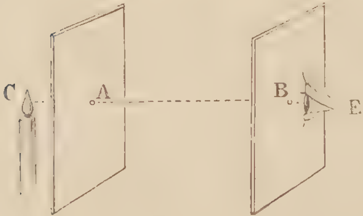

Fig. 2.

bending must take place at the surface of separation of the two media. When, however, a ray of light travels through a non-homogeneous medium, it may suffer gradual and continuous change of direction, if the change in the properties of the medium along its path are also gradual and continuous; it is only on passing from one homogeneous medium to another that sudden change of direction takes place. The magnitude and direction of this change depend on conditions which we shall consider more fully in later chapters.

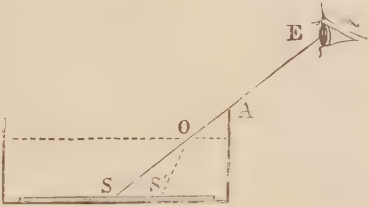

Fig. 3.

It should be noticed in connection with this point that the eye takes no cognisance of change of direction in a ray of light; every object is seen in the direction taken by the axis of the pencil of light which enters the eye. For example, if light, starting from O (Fig. 4), be bent by any means as indicated in the figure, then O appears to the eye to be at O′. The point O′ is the *virtual focus* of the pencil entering the eye, called *virtual* because its rays do not really diverge from O′, but only appear to do so.

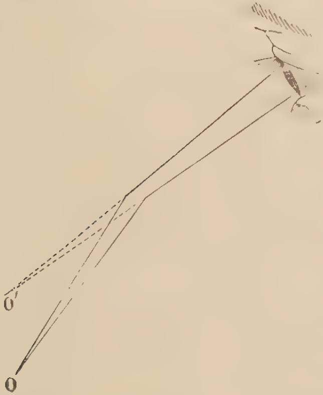

Fig. 4.

8. The pinhole camera. If a sheet of cardboard, pierced, at its centre, with a large pinhole, be placed between a candle and a thin paper screen, shaded from external light, a more or less distinct representation of the candle flame will be seen, in an inverted position, on the screen (Fig. 5).

If the cardboard form the front, and the screen the back of a closed box, the representation can be seen very distinctly from behind, through the paper (or ground glass) screen. The explanation of this is simple. Let A B (Fig. 6) represent the candle flame or other brightly illuminated object, O the hole in the cardboard, and S S the screen. From every point on A B rays are given off in all directions, and consequently from every point of A B a small pencil of rays passes through O, and forms a small circular or elliptical spot on the screen. The result of this is that we have on the screen an assemblage of nearly circular spots, which, owing to the crossing of the rays at O, define an *inverted* representation of the object A B. If these spots are large they overlap one another, and the representation is blurred and indistinct; hence, in order to obtain a well-defined picture of A B on the screen, the aperture at O must be very small; for the size of the spot on the screen depends, for given positions of A B and S S, upon the size of aperture.

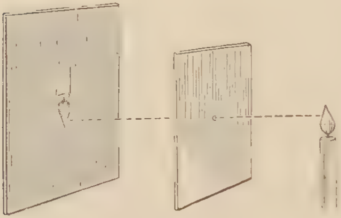

Fig. 5.

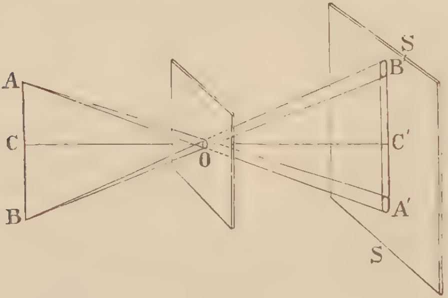

Fig. 6.

It must be noticed that the angle A′OB′ made by the rays after crossing is the same as before (AOB). Therefore AOB, A′OB′ are similar triangles, and AB bears the same ratio to A′B′ as CO to C′O. In other words, size of object, size of image, distance of object, and distance of image form the terms of a simple proportion, so that, if any three of the terms are given, we can easily find a fourth.

Example. Find A′B′ if $OC = 10$ metres, $OC' = 1$ metre, and $AB = 2$ metres. We have

$$\frac{AB}{A'B'} = \frac{CO}{OC'} = \frac{10}{1},$$

$$\therefore\ A'B' = \frac{AB}{10} = 20 \text{ cms.}$$

The circular and elliptical spots seen on the ground beneath trees when the Sun is shining are images of the Sun formed by the rays which have penetrated the interstices of the leaves.

Exp. 1.—**Make a pinhole Camera.** Cut a strip of brown paper 8 ins. by 24 ins. Paste half one side and roll it in the direction of the longer side, with the pasted side inwards, on a roller about 3 ins. in diameter. In this way a serviceable cardboard tube can be made. Over this roll a single layer of dry paper. Over this again roll a pasted sheet of brown paper as before. Thus two cardboard tubes about 8 ins. long and 3 ins. in diameter will be made so that one may slide stiffly in the other. Blacken the insides and cut the ends square. Close one end of the wider tube by a thin piece of card, in the centre of which make a small pinhole. Close one end of the smaller tube with a piece of tracing paper. Push this end of the smaller tube into the larger one, and looking into its open end, direct the pinhole to any brightly illumined object. A small inverted image of the object will be seen on the tracing paper. Push the smaller tube farther in and note the reduced size but increased brightness of the image. Pull it farther out and note the increased size but diminished brightness of the image. Enlarge the pinhole and note the increased brightness but less distinctness of the image, its size being unaltered.

9. Shadows. The formation of shadows is a direct consequence of the rectilinear propagation of light. If an opaque body (B) be placed so as to intercept a portion of the light emitted by a luminous point (L), the cone of light incident on the surface of the body is stopped, and the space beyond, enclosed by the geometrical continuation of this cone, is screened from the rays diverging from L.

The cone here considered (L C C) is called the *shadow cone*, and its trace (S S S S) on any surface intersecting it beyond B outlines the *shadow* cast on this surface. When, however, the source of light is not a luminous point, but a luminous body, the case is somewhat more complicated. Let S S′ (Fig. 8, *a*) represent a spherical source of light, and O O′ an opaque sphere placed near it. Consider the single cone S S′ *u u*, which encloses S S′ and O O′; it is evident from the figure that *no* light from S S′ falls within the portion of this cone lying beyond O O′; and for this reason it has been called the cone of *total shadow*, or the cone of the *umbra*; and the portion of it just referred to as being completely screened from the light is called the umbra. Consider again the double cone S S′A *p p*, enclosing S S′ and O O′, and having its apex at A. This is the cone of *partial shadow*, or the cone of the *penumbra*, and the portion of it beyond

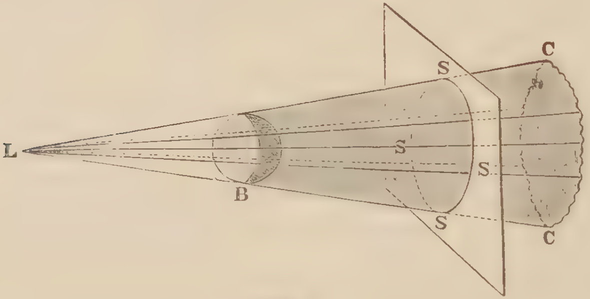

Fig. 7.

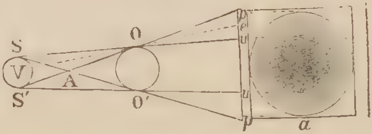

Fig. 8, *a*.

O O′, and surrounding the umbra, is known as the *penumbra*. From any point in the cone, not within the total shadow, a portion of the source of light can be seen, and for this reason the shadow is only partial. The depth of shadow at any point depends on the extent of the source invisible from that point; to an eye placed at *e* all below *eV* is invisible, while all above is visible; hence, at points near the outer boundary of the penumbral cone, the shadow is very light, but gradually deepens as we approach the outer boundary of the umbral cone.

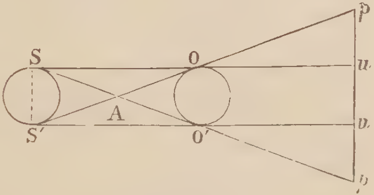

Fig. 8, *b*.

The penumbral cone always has the form of a cone diverging from a point (A) lying between S S′ and O O′, but the form of the umbral cone depends on the relative size of the source of light and the opaque body; when the latter is the greater the cone diverges from a point behind the former (Fig. 8, *a*), when equal it takes the form of a cylinder (Fig. 8, *b*), and when smaller the cone converges to a point beyond the opaque body (Fig. 8, *c*). If the shadow of the opaque body be cast upon a suitably placed screen (Fig. 8, *a*), it will be found to consist of a central region of total shadow, the umbra, surrounded by a zone of partial shadow, the penumbra. The former is of uniform depth all over, but the latter passes gradually from the total shadow of the umbra to a complete absence of shadow at its outer boundary, and consequently neither its outer nor its inner edge is sharply defined. The relative size of the umbra and penumbra in any particular case depends

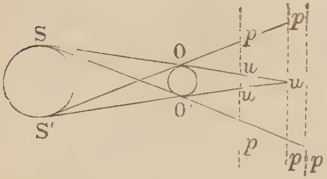

Fig. 8, *c*.

upon the conditions illustrated in Figs. 8, and upon the position of the screen. The reader will find it instructive to draw diagrams for a number of different cases.

Exp. 2. On a narrow gas cylinder or any other convenient support put an opaque ball of any material, about $3\frac{1}{2}$ ins. in diameter. It will be all the better if painted dead black. Behind it, at a distance of about a foot, place a bat's-wing gas-burner with the side of the flame turned towards the ball. 4 or 5 ft. in front of the ball put a piece of card to receive the shadow of the ball. But first turn the burner very low so that the shadow is only faintly visible in the dark room. Although faint, it will be very sharply defined, because with such a small source there can be very little penumbra. Now gradually turn on the gas so as to enlarge the flame. The sharpness will gradually disappear, the penumbra will be very perceptible, and it will be impossible to see exactly where it merges into the perfectly dark umbra on the one hand, and the fully illuminated area on the other. Now vary the distance of the screen. This will only alter the size, not the character, of the shadow, for both umbra and penumbra will enlarge as the distance of the screen increases. Now put over the flame an opal gas globe about 7 ins. in diameter. This whole globe, not the flame itself, will now be the source, and it will be larger than the opaque ball. Accordingly the umbra should get smaller as the distance of the screen increases, and at a certain distance it should terminate, and beyond that distance the shadow should consist of penumbra only. Verify all this by varying the position of the screen.

Exp. 3. Unscrew the tube of a Bunsen burner, and light the gas at the small jet. This will give a long, straight, luminous flame. Adjust it to a height of about 4 ins., and with it cast a shadow of a piece of broomstick. Notice that when the stick is vertical its shadow is fairly sharp, but when horizontal extremely indistinct.

Example. A circular uniform source of light, 2 inches in diameter, is placed at a distance of 10 feet from a sphere 2 inches in diameter. Calculate, approximately, the diameters of the umbra and penumbra cast on a screen 5 feet beyond the sphere.

Here, in Fig. 8, *b*—

$S S' = 2$ inches ; $O O' = 2$ inches ; $S O = 10$ feet ; $O u = 5$ feet.

Diameter of umbra $= u u = O O = 2$ inches.

Diameters of penumbra $\begin{cases} \text{Internal} = u u = 2 \text{ inches} \\ \text{External} = p p = 4 \text{ inches} \end{cases}$; for, from the triangles $O u p$ and $O S S'$, we have, by Euclid vi. 4—

$$\frac{u p}{S S'} = \frac{u O}{O S} = \frac{5}{10} = \frac{1}{2}.$$

But $S S' = 2$ inches. $\therefore \frac{u p}{2} = \frac{1}{2}$, or $u p = 1$ inch.

$$\therefore p p = u u + 2 u p = 2 + 2 = 4 \text{ inches.}$$

10. Eclipses are the result of the shadow of the Earth falling on the Moon (*lunar eclipse*), or of the shadow of the Moon falling on the Earth (*solar eclipse*). In the latter case the Moon is in the position of the opaque body in Fig. 8, *c*. If the Earth is at *uu*, the umbral cone is narrow, as the figure shows. It follows that total eclipses of the Sun are seen from a limited portion only of the Earth's surface. But the penumbra is very broad. A person viewing a solar eclipse from a point in the penumbra sees a portion of the Sun, or a partial eclipse. From places north of the umbra a part of the north border of the Sun is visible, and so on. The Moon appears as a dark object between the observer and the Sun.

If a total eclipse takes place when the Moon is at her greatest distance from us, the Earth is beyond the point of the umbral cone at *u*. From places just behind the apex of the umbra and within the angle formed by producing Ou, $O'u$, an observer sees a narrow ring of the Sun's disc all round, and the central portions are dark. This is called an **annular** eclipse.

In a *lunar eclipse* the Earth is at OO' in Fig. 8, *c*, and the Moon at *uu*. The Moon is seen to pass through the penumbra into the umbra partly or wholly, and then out through the penumbra again. It is evident that we can only have an eclipse when the Earth, Moon, and Sun are in a straight line, or very nearly so. At full Moon the Earth is in the middle, and lunar eclipses are possible. At new Moon the Moon is in the middle, and a solar eclipse may occur. As a matter of fact, eclipses do take place at new and full Moon if the Moon happens to be nearly in the plane of the Earth's motion with regard to the Sun, that is, in the ecliptic.

EXERCISES I.

1. Explain exactly why a small hole is able to produce an image of an object on a screen. What will be the effect of enlarging the hole?

2. A candle is placed inside a box in a dark room. A small hole is cut in one side of the box, and a sheet of paper is held a short distance from the hole. Describe and explain the appearance seen on the paper.

3. In Qu. 2 what would be the effect of cutting two or three small holes on the same side of the box, other arrangements remaining the same?

4. Explain, with diagrams, the conditions under which the shadows of bodies are sharp, and under what conditions they are blurred.

5. Draw a diagram to show that under certain circumstances a shadow may contain no *umbra*, but consist wholly of *penumbra*.

6. A small opaque disc is placed between a gas burner and a white screen, and when the gas is turned down so that the flame is very small it is found that the shadow cast on the screen is quite sharp, but on turning up the gas so that the flame is large that the edge of the shadow is blurred. Explain the reason for this change, illustrating your answer by means of diagrams.

7. Describe a pinhole camera. What experiment would you perform to show why it is that the image of a luminous object first becomes blurred and then disappears when the size of the hole is gradually increased?

8. If a hair be held in sunlight close to a sheet of paper, a sharp shadow is produced. If, however, the hair be held further away, there is no shadow. Explain this.

9. A dark room 10 ft. square, with white walls, has a small hole in one wall. Outside this hole and 56 ft. distant is a stone cross 15 ft. high, and the image appears on the wall of the room. How high will the image be?

10. Under the same circumstances the image of a tree 50 ft. high appears 8 ins. high. How far is the tree from the hole?

11. Draw a diagram of the sun, earth, and moon during an eclipse of the moon. Supposing that the moon traverses the centre of the shadow, find how far off the moon would have to be from the earth in order that the eclipse may be only penumbral.

Diameter of Sun = 880,000 miles.
,, ,, Earth = 8,000 miles.
Distance from Sun to Earth = 92,000,000 miles.

12. A circular uniform source of light, 10 cms. in diameter, is placed 1 metre in front of a spherical opaque body 5 cms. in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it; also find the diameter of the penumbra in this position [Fig. 8, *c*].

13. In Fig. 6, CO = 3 metres, OC′ = 20 cms., and the diameter of the aperture at O is 1 mm. Find the area of the circular spot of light at C′ due to the pencil of light coming from C. If A B = 2 metres, find also the length of A′ B′.

CHAPTER III.

PHOTOMETRY.

11. Sources of Light. Nearly all light is produced either by burning or by incandescence. The earliest domestic sources of light consisted either of wicks immersed in oil (lamps), or of wicks immersed in solid fat (candles). Both these sources are still in use, and have been greatly improved in recent years. Coal-gas was first artificially prepared (by Murdoch) in 1792, and was then only burnt at naked burners. Argand greatly improved the light of both oil and gas lamps by putting a glass chimney around the flame to increase the draught. Bunsen increased the temperature of the flame by mixing air with the gas before burning. This, however, made the flame non-luminous; but if now a body, especially a refractory oxide, is placed in the flame, it is heated to incandescence and gives out a brilliant light. The outcome of this discovery is the Welsbach lamp. The mantle, which is a gauze-like structure, is made up of 99 per cent. thoria and 1 per cent. ceria; the ceria possessing great light-emitting properties. Acetylene gas, owing to its strongly luminous flame and ease of preparation, is also largely used for lighting.

With the advent of electricity, the electric arc and the electric incandescent lamps* were introduced. The former was invented by Davy in 1801, and is admirable for street illumination and optical lantern work, its light being very white. A newer form of the arc lamp uses carbons, impregnated with metallic salts; the arc itself is the source of light in this case, but the light is strongly coloured.

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* See *Higher Text-book of Magnetism and Electricity*, Art. 163.

The ordinary incandescent electric lamp is provided with carbon filaments; in more recent forms the filament is made of osmium or tantalum. The Nernst lamp consists of a filament of certain oxides of rare metals heated to incandescence by an electric current. It is not a vacuum lamp, but has the disadvantage of being rather fragile.

At present there is severe competition between the Welsbach light and the incandescent electric lamp for domestic use, and between compressed gas lamps and electric arc lamps for street illumination.

12. Light as a measurable quantity. Light, like radiant heat, is undoubtedly a form of energy, and, as such, is capable of measurement. The quantity of light in any space at any instant is measured by the corresponding amount of energy available for illuminating bodies in that space at the instant considered, and the physical intensity of the light is measured by the energy transmitted through that space in unit time. For light of a given colour, the intensity conditions the brightness as perceived by the eye (or by a sensitive plate in a photographic camera). The physical intensity is also strictly proportional to the heating effect, produced in a black surface exposed to the light.

There are thus three methods of measuring the intensity of light—the *ocular* or *photometric*, the *photographic*, and the *calorimetric*. In a photographic camera the light does work in producing chemical changes in the salts on the sensitive film. In the calorimetric method a thermopile,* or some similar instrument, is exposed to the radiation, and the energy received is transformed into heat energy, which produces an electrical effect measurable by a delicate galvanometer.

The defect of both the photographic and the calorimetric methods is that they measure other forms of energy physically similar to luminous energy in every respect, except that of being detected by the eye.

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* See *Higher Text-book of Magnetism and Electricity*, § 218, and *Higher Text-book of Heat*, § 97 *et seq.*

In this chapter we shall consider the photometric method only; the other methods will be more fully dealt with in Chapter IX.

13. Intensity of illumination of a surface at a point. Let q denote the quantity of light energy incident per second on a small element of a surface of area s, then the limit of the ratio $\frac{q}{s}$, when s, and therefore q, are indefinitely diminished gives the intensity of illumination of the surface at the point at which s vanishes. If $\frac{q}{s}$ is constant for all points, then the surface is said to be *uniformly illuminated;* and if Q denote the quantity of light energy incident per second upon a portion of the surface of area S, then $\frac{\mathrm{Q}}{\mathrm{S}}$ determines the intensity of this uniform illumination.

14. Law of inverse squares.

Exp. 4. Cut out a hole 3 ins. square in a piece of brown paper or cardboard. Fix it one foot from a wall and parallel to it. At two feet from the wall and opposite the hole place a candle. The light going through the hole will give a square patch of light on the screen. Measure its dimensions. It is approximately a 6-in. square.

Deduction. The same amount of light that illuminates the 3-in. square illuminates the 6-in. square at double the distance. But the area of the 6-in. square is four times that of the 3-in. square. Therefore the intensity of illumination of any point in the 6-in. square is only one-quarter that of any point in the 3-in. square, *i.e.* when the distance from the source is doubled the intensity of illumination is reduced to one-fourth.

Repeat for various distances, also with different shaped apertures, and from your results formulate a law. The law you will obtain is called the law of inverse squares.

It states that *the intensity of illumination, of any uniformly illuminated surface, is inversely proportional to the square of its distance from the source of light;* or, in more general terms, *the intensity of illumination, at any* *point of a surface, is inversely proportional to the square of the distance of that point from the source of light.*[*]

To prove the general case let P (Fig. 9) represent a luminous point, and let us conceive this point to be surrounded by a sphere $S_1 S_1 S_1$ of radius, R_1, having its centre at P. Then the inner surface of this sphere will be uniformly illuminated, and if no light be absorbed by the medium the intensity of illumination, I_1, is given by—

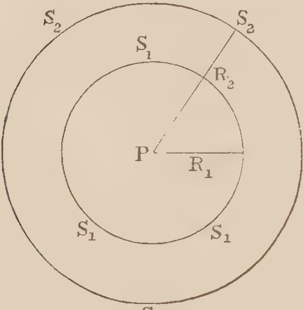

Fig. 9.

$$I_1 = \frac{Q}{4\pi R_1^2}\ [†]$$

where Q denotes the total quantity of light energy emitted by P per second. Similarly, if we consider the sphere $S_2 S_2 S_2$, the intensity of illumination of its inner surface is given by—

$$I_2 = \frac{Q}{4\pi R_2^2}.$$

Hence we have—

$$\frac{I_1}{I_2} = \frac{Q}{4\pi R_1^2} \Big/ \frac{Q}{4\pi R_2^2} = \frac{R_2^2}{R_1^2}.$$

For an experimental proof of this law see Exp. 6.

It follows from the above that if I_1 denote the intensity of illumination of a surface at unit distance from the source of light, and perpendicular to the rays, then the intensity of illumination I_0 on a similarly placed surface at a distance R is given by—

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[*] In this and the following articles the dimensions of the source of light are considered to be so small, compared with the other distances involved, that it may be treated as a luminous point. If the source of light have a large surface, the illumination for points close up or near to the surface is practically independent of the distance.

[†] The area of the surface of a sphere of radius $R = 4\pi R^2$.

$$I_0 = \frac{I_1}{R^2}, \quad \text{for } \frac{I_0}{I_1} = \frac{1}{R^2}\,; \text{ that is, } I_0 = \frac{I_1}{R^2}.$$

Example. The intensity of illumination of a screen placed 6 feet from a given source of light is denoted by I. Find the intensity when the distance of the screen is increased to 9 feet.

Let I′ denote the required intensity. Then, by Art. **14**—

$$\frac{I'}{I} = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

That is, $I' = \frac{4}{9}I$.

15. The intensity of illumination of a surface varies with the angle of incidence of the light. The angle of incidence is the angle made by the axis of the incident pencil of light with the normal to the surface. Let P A B (Fig. 10) denote a pencil of light emanating from P, and incident on the surface A B at an angle represented by P O N. If Q denote the quantity of light energy emitted per second by P in the pencil P A B, the intensity of illumination of $AB = \frac{Q}{\text{area of } AB} = \frac{Q}{A} = I.$

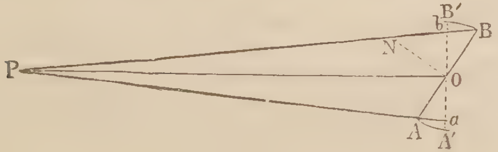

Fig. 10.

Now imagine the surface A B to be rotated round O into a position A′ B′, where it is at right angles to P O. The same quantity of light now falls on the smaller area, *ab*, and the intensity of illumination of this area $= \frac{Q}{\text{area of } ab} = \frac{Q}{A_0} = I_0.$

It can, however, be shown (Art. 18) that $A_0 = A \cos BOB' = A \cos PON$, assuming that the area A B is very small, and the angle B b O thus very nearly a right angle.

Therefore we have—

$$\frac{I}{I_0} = \frac{Q}{A} \Big/ \frac{Q}{A_0} = \frac{A_0}{A} = \frac{A \cos PON}{A} = \cos PON.$$

That is, $I = I_0 \cos PON$.

This result may be expressed thus:—If I_0 denote the intensity of illumination of a given surface when the light is incident normally, then, for incidence at an angle θ, the intensity of illumination is given by $I = I_0 \cos \theta$; that is, *the intensity of illumination varies directly as the cosine of the angle of incidence.*

This partly explains why the strength of the sun's heat and light is greater on fields and hillsides which lie directly facing the sun, and less upon places which slope away from the sun, and so lie obliquely to the rays.

Now since $I_0 = I_1/R^2$ and $I = I_0 \cos \theta$ we have $I = \frac{I_1 \cos \theta}{R^2}$, which gives the intensity of illumination of a surface placed at a distance R from the source of light, the angle of incidence being θ, and the source being of such power that the intensity of illumination of a surface placed at unit distance from the source, and perpendicular to the rays, is I_1.

16. The illuminating power of any source of light. The intensity of any source of light is proportional to its illuminating power. *This quantity is measured by the intensity of illumination of unit area of a surface placed at unit distance from the given source, the light being incident normally on this surface.* If L_1 denote the illuminating power of a given source of light (A), then $L_1 = I_1$, and the intensity of illumination produced by this source on a surface, at a distance R_1 when the angle of incidence of the light is θ_1, is given by—

$$I = \frac{L_1 \cos \theta_1}{R_1^2}. \quad \text{(Arts. 14, 15.)}$$

Similarly, if another source of light (B) be so placed as to produce the *same* intensity of illumination (I) for an angle of incidence of the light θ_2, then—

$$I = \frac{L_2 \cos \theta_2}{R_2^2}$$

where L_2 denotes the illuminating power of B, and R_2 its distance from the illuminated surface. Now, equating these two expressions for I, we have—

$$\frac{L_1 \cos \theta_1}{R_1^2} = \frac{R_2 \cos \theta_2}{R_2^2}.$$

This is the general formula. Usually in the comparison of sources of light it is arranged that $\theta_1 = \theta_2$.

In this case—

$$\frac{L_1}{R_1^2} = \frac{L_2}{R_2^2} \text{ or } \frac{L_1}{L_2} = \frac{R_1^2}{R_2^2}.$$

This shows that *the illuminating powers of different sources of light are* directly* *proportional to the squares of the distances they must be placed from a given surface, in order to produce on it the same intensity of illumination* It should be noticed that the above is true only when the angle of incidence, θ, is the same in each case; hence it is evident that in an experimental comparison of illuminating powers care must be taken to satisfy this condition.

Example. Two sources of light, A and B, when placed respectively 8 and 10 feet from a screen, produce the same intensity of illumination of its surface. Compare the illuminating powers of A and B.

Here, by Art. **16**—

$$\frac{\text{Illuminating power of A}}{\text{Illuminating power of B}} = \left(\frac{8}{10}\right)^2 = \frac{16}{25}.$$

Note. It is important to distinguish between the quantities *intensity of light*, *illuminating power*, and *intensity of illumination*. Intensity of light is a quantity involving energy (Art. **12**); illuminating power of a source of light is proportional to the intensity of the light, and is measured as stated in Art. **16**; intensity of illumination refers to the surface illuminated and not, like the other two quantities, to the source of light (Arts. **13-15**).

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* This statement must be carefully distinguished from that made at the beginning of Art. **14**.

17. Photometers. Photometry is the experimental comparison of the illuminating powers of different sources of light, and the different forms of apparatus by which this comparison is effected are called *photometers.* The practical unit employed in photometry is the light emitted by a **standard sperm candle**, six to the pound, burning 120 grains per hour; and hence the illuminating power of any source of light is generally expressed as being equivalent to that of a certain number of standard candles. It has been found that the eye is unable to estimate the ratio of the intensity of illumination due to different sources of light, but that it is a better judge of the equality of the illumination of two adjacent surfaces. For this reason nearly all methods of photometry depend on the equalisation of the illuminations of two adjacent white screens, and the details of the construction of photometers are devised to facilitate this adjustment. All photometrical experiments must be made in a dark room.

Foucault's photometer. This photometer (Fig. 11) consists of a semi-transparent screen, A B, of thin paper, ground glass, or thin white porcelain, fixed vertically in front of, and at right angles to, a partition, C D, which is movable by means of a screw in the direction of its length. The two sources of light to be compared, L_1 and L_2, are placed on opposite sides of this partition in such positions that the angle $L_1 E L_2$ is bisected by C D. By this arrangement one portion of the screen is illuminated by one source

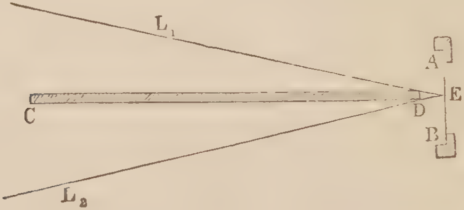

Fig. 11.

and the other portion by the other; and, by adjusting the position of C D until these separately illuminated portions become contiguous, their illuminations may be more accurately compared.

When both portions appear equally bright the comparison is complete, and we have, if L_1 and L_2 represent the final positions of the sources of light—

$$\frac{\text{Illuminating power of } L_1}{\text{Illuminating power of } L_2} = \frac{(E\,L_1)^2}{(E\,L_2)^2}.^*$$

Example.—Find the candle-power of a gas flame which, if placed three times as far away from a surface as a standard candle, illuminates it to the same extent.

We have—

$$\frac{\text{Power of gas flame}}{\text{Power of candle}} = \frac{3^2}{1^2} = 9.$$

∴ Gas flame is 9 candle-power.

Rumford's photometer. In this photometer the illuminating powers of two sources of light are compared by adjusting to equality the intensities of the two shadows of a vertical rod cast on a screen by the two given sources. The lights (L_1, L_2), rod (R), and screen (S S) are arranged, as shown in Fig. 12, so that the two shadows (S_1x, S_2x), whose edges should be well defined, appear close together, and of equal intensity. In this way, **since each shadow is illuminated by the source to which the other is due, equality of intensity of the shadows cast by L_1 and L_2 means equality of illumination due to L_2 and L_1**; and hence we have, as in the previous case—

$$\frac{L_1}{L_2} = \frac{(L_1\,x)^2}{(L_2\,x)^2}.$$

Note. Since the shadows must be purely umbral, the sources of light must be very narrow. Also L_1x_1, L_2x_2 must be equally inclined to the screen.

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* The dimensions of the screen are small compared with the distances $E\,L_1$ and $E\,L_2$.

Exp. 5. Set up a simple Rumford photometer; a small white card on a black-board is an effective screen; for the stick use a pencil or the rod of a retort stand. Darken the room. Take two sources of light—a candle and a lamp with a narrow flame or two candles of different sizes. Mount them, and find several pairs of positions in which the shadows are alike. Find the value of $\frac{L_1}{L_2}$ for each pair of positions. The result is nearly constant. The mean is the ratio between the illuminating powers of the two lights.

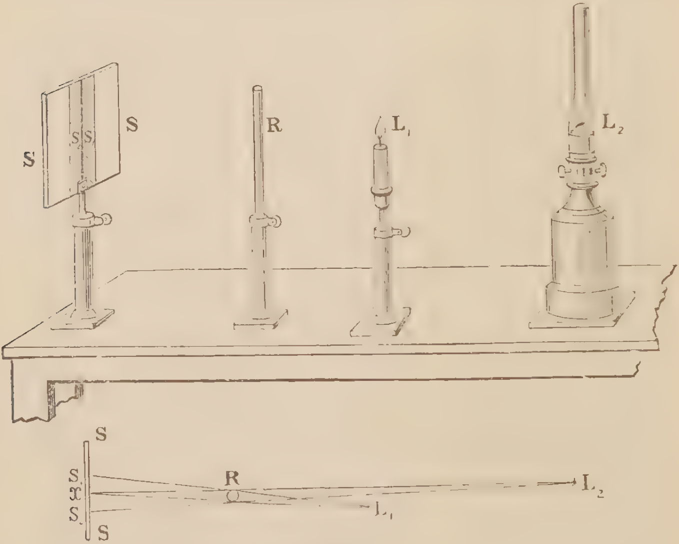

Fig. 12.

Bunsen's photometer. Bunsen devised a very simple form of photometer. If a sheet of paper having a spot of grease on it be held up to the light, it will be seen that the spot of grease is semi-transparent, and looks brighter than the rest of the paper when viewed from the side remote from the light, but darker when seen from the other side. The reason of this is evident; more light passes through the region of the grease spot than through the rest of the paper, and hence when seen from the side remote from the light it looks brighter than the rest of the screen, through which little or no light passes; when looked at on the other side, however, the spot looks comparatively dark, because a large proportion of the light incident upon it passes through, and is therefore not spent in illuminating its surface. It will now be understood that if a suitable paper screen, having a grease spot at its centre, be placed between two lights, A and B, and its position adjusted until the spot cannot be seen on either side, except by close inspection, then the screen must be equally illuminated on both sides. For, if the grease spot is not readily distinguishable from the adjacent surface of the screen, the amount of light coming from unit area of both must be the same. Let Q denote the quantity of light incident from A on unit area of the surface of the screen, and q the quantity that passes through, per unit area, in the region of the spot. Then $Q—q$ denotes the quantity of light spent in illuminating the surface of unit area on the grease spot; whereas the whole quantity,* Q, is spent in illuminating the surface of unit area of the screen in the neighbourhood of this spot. Hence, on the side of A, the spot will appear dark unless a quantity of light, q, is transmitted through it from B to make up for the quantity that has passed through from A. It thus appears that a necessary condition for the disappearance of the spot is that the quantities of light passing through it in opposite directions must be equal. But, if the surface of the spot is the same on both sides, the quantity of light that passes through will be, for each side, the same fraction of the light incident on that side; and consequently, if the quantities of light passing through the unit area of the grease spot are equal, then the quantities of light incident on unit area of opposite sides of the screen are also equal*—that is, the two sides of the screen are equally illuminated. If L_1, L_2, and S S (Fig. 13) represent the relative

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* To simplify matters the screen is considered to be opaque, except at the grease spot, and its surface to be such that there is no regular reflection. The general case is easily treated, and leads, in a similar way, to the same result.

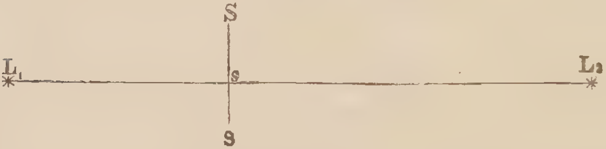

Fig. 13.

positions of the lights and the screen when finally adjusted, we have—

$$\frac{L_1}{L_2} = \frac{(L_1\,s)^2}{(L_2\,s)^2}.$$

In practically carrying out the necessary measurements at least four different adjustments should be made: (1) Adjust for disappearance of the spot when seen from the side of screen facing L_1. (2) Turn the screen round through 180°, and again adjust for disappearance of spot from the same surface now facing L_2. (3) Repeat (1) and (2) with the other surface of the screen.

The screen is usually mounted in a light frame, so that it can be easily turned round in its stand, or as a whole.

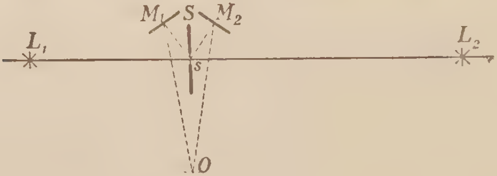

Fig. 14.

The stand also often carries two mirrors M_1, M_2, arranged as in Fig. 14, so that an observer at O can view both sides of the grease spot at the same time and nearly under the same conditions.

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* That is, if $q = \frac{Q}{n}$, then, when q and n are the same for both sides of the screen, Q must also be the same for both sides.

Joly's photometer. This consists of two equal blocks of paraffin wax, W_1, W_2 (Fig. 15), about a quarter-inch thick separated by a smooth thin sheet of tinfoil, T, and mounted on an open wooden slider, B B. The light coming from either source enters the wax and is scattered both before and after reflection from the surface of the tin. The tin is opaque, so that W_1 is illuminated solely by L_1 and W_2 by L_2. In taking a measurement the compound block is moved up and down the line joining the luminous points until a position is found in which W_1 and W_2 appear equally bright to an observer looking at them through a hole in B B. The position of T is then read and we have—

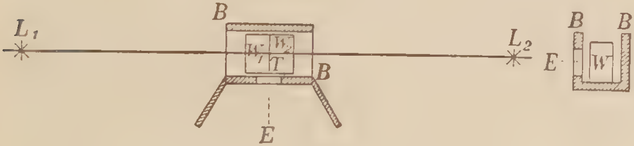

Fig. 15.

$$\frac{L_1}{L_2} = \left(\frac{L_1T}{L_2T}\right)^2.$$

Exp. 6.—To make a grease-spot photometer and to prove the law of inverse squares. Cut a hole 2 ins. square near one end of a piece of cardboard about 3 ins. by 6 ins. Make a grease spot (in the shape of a star) on a piece of white unglazed paper and then paste the paper over the hole so that the star is central. Fix the card vertically.

(1) Take two equal candles, trim, mount, and light them, and move them up and down the line L_1sL_2 (Fig. 13), until the grease spot and the rest of the paper are equally bright. If the candles are burning equally, they should then be equally distant from the screen. Confirm.

(2) Mount four equal candles on a support, and place them so that the four are parallel to the screen. Compare them against the single candle on the other side. In observing, look alternately at either side of the screen until the appearance is the same on both

sides. If possible, arrange two mirrors so as to be able to see both sides at once. Screen off all exterior light. To get an exact result move the single candle to and fro in the line $L_1 s L_2$ so that in one position the spot is too bright and in the other too dark. By making the candle *oscillate* between these two positions, and gradually narrowing down the extent of the oscillations, the position of the candle can be determined with great exactness. Enter up readings in a table as shown. The last column is very nearly constant and equal to four. Therefore one candle at a certain distance illuminates the screen as much as four candles at twice this distance. Hence the law of inverse squares. Repeat for different numbers of candles.

Distance of single candle from screen. l_1	Distance of the four candles from screen. l_4	Ratio $\left(\frac{l_4}{l_1}\right)^2 = L_4/L_1$.

In all photometric measurements the lights to be compared should be of the same colour. If this is not the case it will be found impossible to adjust accurately for equal illumination, owing to the difference in colour of the illuminated surfaces. In general, different sources of light emit differently coloured rays in different proportions (Art. 93), so that an accurate comparison of intensity can only be made by means of an instrument which forms the light from each source into a spectrum (Art. 95), and admits of a comparison of corresponding parts of the spectra so formed.

Candle-power of different sources of light. The following table gives the candle-power of some common and other sources of light :—

Ordinary gas jet	10-18
Welsbach burner and mantle ...	45
Common Argand burner	11-17
Electric glow lamp	16 (usual house size)
Electric arc lamp	1,000-2,000
Lamp on the Eddystone	80,000
Lamp on Belle Isle (Finisterre)	30,000,000

The light given out by an ordinary candle contains only 2 per cent. of the total energy consumed. The ratios for electric incandescent and arc lights are 7 and 15 per cent. respectively. The light given out by the sun carries 35 per cent. of the total output of energy, while that given out by the Cuban firefly carries 99 per cent., so that, in comparison with Nature, man's appliances are not economic.

18. Plane Angles.* In this chapter we have made use of the term *cosine*, and in succeeding chapters it will be necessary to make frequent use of the terms *sine* and *tangent*. Hence for the convenience of the reader we shall now explain these terms.

Let D A E represent a plane angle. From any point C, in A E, draw C B perpendicular to A D, and cutting A D in B. Now the length of B C, for a given position of C, evidently depends on the magnitude of the angle D A E, but it gives no indication of this magnitude unless the position of C be defined. For this purpose the ratio $\frac{BC}{AC}$ may be considered, and it can be shown geometrically that wherever C be taken on A E this ratio is constant, and is definitely related to the magnitude of the angle B A C. Similarly the ratio $\frac{AB}{AC}$ is constant and bears a fixed relation to the magnitude of B A C. The ratio $\frac{BC}{AC}$ is called the *sine* of B A C, the ratio $\frac{AB}{AC}$ is called the *cosine* of B A C, and the ratio $\frac{BC}{AB}$ is called the *tangent* of B A C. In the right-angled triangle B A C, *considered with reference to the angle* B A C, the side B C is called the *perpendicular*, the side A B is called the *base*, and A C is called the *hypotenuse*. Hence, in general terms—

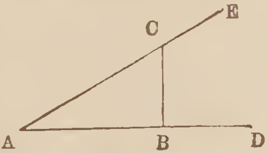

Fig. 16.

$$\text{sine BAC} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \sin \text{BAC}.$$

$$\text{cosine BAC} = \frac{\text{base}}{\text{hypotenuse}} = \cos \text{BAC}.$$

$$\text{tangent BAC} = \frac{\text{perpendicular}}{\text{base}} = \tan \text{BAC}.$$

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* This section may be postponed until Exercises II. have been worked.

The reader should deduce geometrically the values of these ratios for angles of 30°, 45°, and 60°. These will be found to be—

$$\sin 30° = \frac{1}{2}. \qquad \cos 30° = \frac{\sqrt{3}}{2}. \qquad \tan 30° = \frac{1}{\sqrt{3}}.$$

$$\sin 45° = \frac{1}{\sqrt{2}}. \qquad \cos 45° = \frac{1}{\sqrt{2}}. \qquad \tan 45° = 1.$$

$$\sin 60° = \frac{\sqrt{3}}{2}. \qquad \cos 60° = \frac{1}{2}. \qquad \tan 60° = \sqrt{3}.$$

Inverse Functions. The angle B A C is the angle whose *sine* is equal to the fraction $\frac{BC}{AC}$. This is often written $\angle BAC = \sin^{-1}\frac{BC}{AC}$. Similarly $\angle BAC = \cos^{-1}\frac{AB}{AC} = \tan^{-1}\frac{BC}{AB}$. For example, $30° = \sin^{-1}\frac{1}{2} = \cos^{-1}\frac{\sqrt{3}}{2} = \tan^{-1}\frac{1}{\sqrt{3}}$.

The magnitude of a Plane Angle may also be expressed in **Circular Measure.**

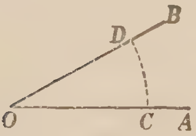

Fig. 17.

Let A O B be a plane angle and C D an arc struck with radius O C. Then the fraction $\frac{\text{arc } CD}{\text{radius } OC}$ is called the circular measure of the angle A O B, and is generally denoted by θ. We have therefore

$$\text{arc} = \text{radius} \times \text{circular measure of angle}$$
$$= r\,\theta.$$

Thus, the circular measure of the angle subtended by a semicircle at the centre of the circle (*i.e.* of an angle of 180°)

$$= \frac{\text{semi-circumference}}{\text{radius}} = \frac{\pi r}{r} = \pi.$$

Similarly the circular measure of $90° = \frac{\pi}{2}$.

The reader will note that when the angle A O B is *very small*, the ratio $\frac{\text{arc } CD}{OC}$ is very nearly equal to either $\frac{CE}{OC}$ or $\frac{FD}{OC}$, *i.e.* to either the tangent or sine of the angle A O B. This may be expressed as $\theta = \tan\theta = \sin\theta$, also in the same case $\cos\theta = \frac{OF}{OD}$, and is very nearly equal to unity.

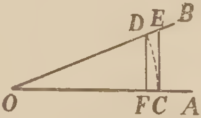

Fig. 18.

EXERCISES II.

1. Explain the principle of the Bunsen (or grease-spot) photometer. How would you prove that the illumination of any surface is inversely proportional to the square of the distance from the source of light?

2. What do you understand by the intensity of illumination at a point due to a given source?

3. Explain how to compare the intensities of two sources of light by means of the shadow photometer.

4. One of two gas-flames gives out twenty-five times as much light as the other. If you are comparing their illuminating powers by means of a Rumford's photometer, and you place the smaller flame at a distance of 2 feet from the screen, at what distance must the larger flame be placed in order that the shadows of an opaque object cast by the two flames may be equally illuminated?

5. The illuminating powers of two sources are in the ratio 4 : 9 If these sources are 200 cms. apart, where would a Bunsen's photometer be in accurate adjustment between them?

6. With a Rumford photometer the shadows of the rod thrown by a bat's-wing gas-flame and a Welsbach incandescent gas-light are equally bright when the bat's-wing is 2 feet from the screen and the Welsbach 4 feet 3 inches. How many times more light does the latter give than the former?

7. The grease-spot of a Bunsen photometer disappears when the standard candle-flame is 10 inches from one side and an electric glow-lamp 36 inches from the other side. What is the candle-power of the lamp?

8. If a 16-candle-power gas-flame at a distance of 10 feet illuminates a surface to a particular degree of brightness, at what distance must a 20-candle-power electric glow-lamp be placed from that surface to illuminate it to the same degree?

9. Three standard candles are placed 10 inches from one side of the screen of a Bunsen photometer. How far must a 5,000-candle-power electric arc be placed from the other side in order to cause the disappearance of the grease-spot?

10. A lighted candle is held (1) in front of, (2) behind a sheet of white paper in the centre of which a grease stain has been made. Explain what you would see on looking at the paper in each case.
The same sheet is now held vertically between two lighted candles, and in a certain position the grease stain cannot be detected by the eye. Why is this?

11. The lines joining the points A, B, and C form an equilateral triangle. D is the middle point of BC. A screen is placed at A with its surface parallel to BC. Lights placed at B, C, and D are found to equally illuminate the screen at A; compare their illuminating powers.

12. The distance between two incandescent lamps of 16 and 25 candle-power respectively is 6 feet. Show that there are two positions, on the line joining the lamps, at which a screen may be placed so as to receive equal illumination from each lamp; and determine these positions.

EXAMINATION QUESTIONS.

1. Explain the phenomena of shadows, and give a sketch illustrating the formation of *umbra* and *penumbra*.

2. A hole is bored in the shutter of a darkened room, and the light transmitted through it is allowed to fall on a whitewashed wall. Describe and explain the appearances presented.

3. Describe experiments to prove that light proceeds in straight lines.

4. When the light of the sun passes through the small spaces between dense foliage, it is noticed that the spots of light formed on the ground are always circular or oval and never angular. Why is this?

5. Define the terms *intensity of illumination* and *illuminating power*. How may the illuminating powers of two sources of light be compared by means of Bunsen's photometer?

6. When a surface is illuminated by a small source of light, how does the illumination of the surface depend on its distance from the light? Does the law hold in the case of a large source of light?

7. Rays of light from a bright gas flame pass through a small pinhole in a black screen, and are received on a sheet of ground glass. Describe by the help of a picture the image seen on the glass. What would be the effect of making the pinhole square instead of round?

8. Show how a penumbra is formed round the edge of a shadow if the source of light has appreciable size.

Draw a careful figure showing the regions of full shadow, and of penumbra, when an obstacle 1 inch in diameter is placed in front of a source of light 2 inches in diameter.

9. If a white paper screen is held a short distance from a hole in a shutter of a darkened room, a picture of the outside view is seen on the screen. Explain how it is formed.

10. How do you account for the relation which exists between the amount of light falling on a given surface and its distance from the illuminating source?

Describe the grease-spot photometer, and explain how you would use it to verify the relation referred to above.

11. Describe carefully any experiment in which the illuminating power of a gas flame can be compared with that of a candle. Illustrate your answer by numbers taken from a supposed experiment.

12. Define the candle-power of a source of light, and describe carefully some one way of measuring it. Must the size of the source be taken account of in the method you describe?

13. What is meant by the statement that a certain lamp is of 16 candle-power? How would you test the accuracy of the statement?

It is found that a 16 candle-power lamp produces the same intensity of illumination on a sheet of paper 8 feet from it as an arc lamp which is 60 feet from the paper. Find the illuminating power of the arc lamp.

14. A pencil is fixed vertically 6 inches in front of a vertical white screen. Three sources of light, A, B, C, of 16, 18, and 48 candle-power, are placed at distances of 2, 3, and 4 feet respectively from the screen, and are so arranged that the three shadows of the pencil thrown by them are close together but do not overlap. Compare the relative degrees of illumination of the shadows.

15. Explain the construction and use of the Bunsen, or grease-spot, photometer.

Two sources of light, A and B, placed 100 cms. apart, lie on either side of the screen of the photometer, A having four times the power of B. Find the position of the screen so that its sides shall be equally illuminated.

If a semi-transparent sheet, which cuts off $\frac{5}{9}$ of the light falling upon it, is placed immediately in front of the source B, in what direction and how far must the photometer screen be moved in order that its sides may be equally illuminated?

16. A source of light is placed 3 feet from a screen. Just in front of the light is placed an opaque disc which can be rotated about a horizontal axis through its centre. One quarter of the disc is removed by cuts along two radii at right angles and except when the gap so formed in the disc is between the light and the screen the rays from the source are intercepted. Compare the illumination of the screen when the gap in the disc is stationary between the source and the screen with that when the disc is rotating rapidly.

Through what distance must the screen be moved in the second case in order that the illumination may be the same as it was originally in the first case?

CHAPTER IV.

REFLECTION AT PLANE SURFACES.

19. The Incident and Reflected Rays.

Exp. 7. Take a clean polished rectangular piece of thin glass* or mirror, M M, and mount it on a rectangular piece of wood so that when the wood is placed on a sheet of cartridge paper pinned to a drawing board the mirror stands vertical. Mark its trace M M (Fig. 19). Place two pins, J and S, almost anywhere in front of the mirror so that the line joining them cuts the mirror, and looking into the mirror move the eye about until the images of J and S in the mirror look in line. Then stick two more pins, Q and R, in the paper so as to be in a line with these images. It is now evident that of the rays of light proceeding from J, one ray going in the direction J S would after reflection at the mirror take the direction Q R. Place the eye now beyond J, and observe that J and S are also in line with the images of Q and R. Remove the mirror and pins. Join J S and Q R, and produce them; they will intersect at a point P on M M. Draw P N at right angles to M M, and compare by compasses or a protractor the magnitudes of the angles J P N, R P N. What do you observe? The angles are evidently equal.

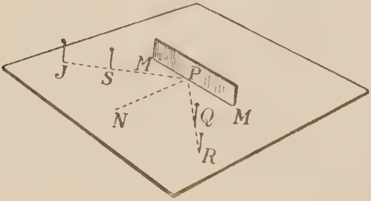

Fig. 19.

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* An ordinary piece of mirror is not very suitable, as it has two reflecting surfaces; if, however, the backing of the mirror be dissolved off with alcohol, and the metallic surface polished with rouge, a very good reflecting surface is obtained.

Repeat with the pins and mirror in different positions, the same result always follows.

In any one case of the above place the eye nearly on a level with the paper, and observe that the positions of the bases of the pins may also be made to exactly cover one another, thus showing that the bases of the pins and their images are all in the plane of the paper, which is perpendicular to the mirror.

20. When a ray of light travelling in one medium, A, is incident on the surface of another medium, B, it is, in general, broken up into three parts:

1. A portion which is reflected from the surface of B, back into A, according to a certain law. This portion is said to suffer reflection at the surface of B in accordance with the laws of reflection.
2. A second portion passes into B, and travels through that medium in a direction determined by another law. This portion is said to be refracted into the medium B in accordance with the laws of refraction.
3. A third portion is scattered or diffused by the surface both into B and A in an irregular manner. The light thus scattered renders the surface luminous, and it is because of this scattering of light by the surfaces of non-luminous bodies that they become luminous in the presence of a self-luminous body (Arts. 2 and 33).

When light is incident upon an opaque body no portion of the light is refracted or diffused into it, and the ratio of the quantities reflected and diffused back depends on the nature of the surface of the body and on the angle at which the light falls on the surface. A rough, uneven surface scatters the greater portion of the light falling on it; but a smooth, highly polished surface reflects nearly all the incident light; also the more obliquely light falls upon any reflecting surface the greater is the proportion of reflected light. Since it is the scattered or diffused light from a surface which renders the surface visible, it follows that a perfectly reflecting surface would be invisible.

21. Mirrors. Any good reflecting surface is a mirror. The term is, however, usually confined to polished surfaces of a definite geometrical form—*e.g.* plane, spherical, cylindrical, etc. The oldest mirrors were of polished metal, and this form of reflector is still in use for optical purposes. The ordinary plane mirror consists of a sheet of plate glass backed by a thin layer of deposited silver, which forms the reflecting surface.* More recently, for scientific purposes, silver *specula* have been employed as mirrors. These are formed of glass surfaces, of the required geometrical form, coated in front with a thin layer of silver which is very highly polished.

22. Definitions. The **normal** to a reflecting surface at any point is a line drawn at right angles to the tangent plane to the surface at that point. If the surface is plane, then the normal at any point is at right angles to the surface; and if spherical it is coincident in direction with the radius drawn to that point. The **angle of incidence** of a ray falling on the surface of a medium is the angle between the direction of the ray and the normal to the surface at the point of incidence. The **plane of incidence** is the plane containing the normal and the incident ray. The **angle of reflection** is the angle made by the reflected ray with the normal at the point of incidence. The **plane of reflection** is the plane containing the normal and the reflected rays.

At this point we may mention what is called the **reversibility of light**. It may be stated thus: If by any means light is able to travel from a source at A to a point B, then, if the source is placed at B, light will travel back to the point A by the same path.

23. Laws of Reflection. When a ray of light is incident on a reflecting surface, it is reflected in accordance with two laws, which may be enunciated as follows:

1. The angle of reflection is equal to the angle of incidence. (NPR = NPI, Fig. 20.)

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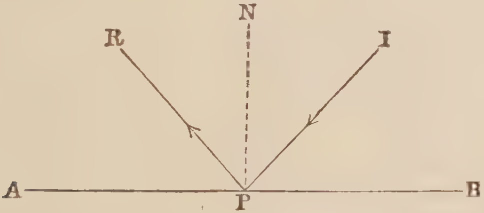
* Up to 1840 all glass mirrors were backed with an amalgam of mercury and tin. Since then this process has been almost entirely superseded by the silver process.

2. The planes of incidence and reflection are coincident.

These may be expressed as one law thus: The angles of incidence and reflection are in the same plane, and are equal to one another.

This law is established by Exp. 7 and is verified by Exp. 9.



A B, Reflecting surface.	I P, Incident ray.
P N, Normal.	P R, Reflected ray.
I P N, Angle of incidence.	N P R, Angle of reflection.

Fig. 20.

The strongest proof of these laws, however, lies in the fact that in numerous experiments the above laws are assumed, and not once has the assumption led to an inaccurate result.

The laws hold for any smooth surface, whether plane or curved. If curved a small area around the point of incidence will be coincident with the tangent plane at that point, and the normal can be drawn perpendicular to this plane.

Example. The sun's rays are inclined at an angle of 30° to a plane mirror. What is the angle of incidence, and what is the angle between the incident and reflected beams?

Using Fig. 20 the $\angle BPI = 30°$, therefore $\angle NPI = 60°$ and this is the angle of incidence.

The angle between the incident and reflected beams is the $\angle IPR$. Obviously it is 120°.

24. Images. When a luminous body is viewed directly, pencils of light from every point on the body enter the eye, and thus the body is seen and its form defined. If, however, from any cause these pencils suffer change of direction, such that they actually come from. or appear to come from, an assemblage of luminous points other than the luminous surface of the body, this assemblage of luminous points is called the *image* of the luminous body. An image may be either *real* or *virtual*; in a real image the rays actually do pass through the points of the image, but in a virtual image they only appear to do so, or perhaps it is better to say that a virtual image is such that the rays are straight lines whose directions would pass through it, if produced backward through the reflecting surface. A real image differs from a luminous body in the fact that the latter emits light in all directions, whereas the former transmits light only in the direction taken by the rays involved in its formation.

25. Parallax. The apparent change in the position of an object due to change in the point of view of the observer is called *parallax*.

Exp. 8. Stand a yard or so behind the centre bar of the frame work of the window and looking out through the window alter the position of the head until this bar appears to "cover" some object out of doors, say a telegraph pole. Now move the head to the right. Observe that, relative to the window bar, the pole has moved to the right. Now move the head to the left; observe that, relative to the window bar, the pole has moved to the left. In each case there has been parallax, and the same always follows with two objects situated at different distances from the observer, viz. relatively to the nearer object the far one moves in the same direction to the observer, and relatively to the farther object the near one moves in the opposite direction to the observer.

A moving railway train affords scope for observations of the same nature.

Errors due to parallax must be carefully guarded against when reading the length of a line by means of a scale, the graduations of which are on the upper side of the scale. As the eye is moved along over the scale the reading of a point on the paper appears to alter, and to take an exact reading the line joining the eye to the point on the line under observation should be perpendicular to the paper.

If two objects are coincident there is no parallax between them, hence if it is required to alter the positions of two objects—or an object and an image—until they are coincident, all that is to be done is to alter their positions until the parallax observed when the eye is moved decreases to zero.

Exp. 9. **To show by the parallax method that the image of an object due to a plane mirror is as far behind the mirror as the object is in front.**

Take the piece of mirror or thin glass of Exp. 7, stand it in a vertical position upon a sheet of cartridge paper pinned to a drawing-board and mark its trace M R (Fig. 21). Stick a pin, A B, vertically into the paper about four inches in front of it; an image A′B′ of the pin, formed by the nearest or polished surface of the glass, will be clearly seen on the opposite side of the plate. Place another pin, C D, behind the mirror so as to coincide in position with this image for all positions of the eye; *i.e.* until all parallax is eliminated. Remove mirror and pins, join B D cutting M R in N, and, by direct measurement with a pair of compasses and a scale, show that (1) the distances of the image and the object from the reflecting surface are equal, *i.e.* B N = B′ N, and (2) that the line B N B′ joining them is perpendicular to the front of the reflecting surface.

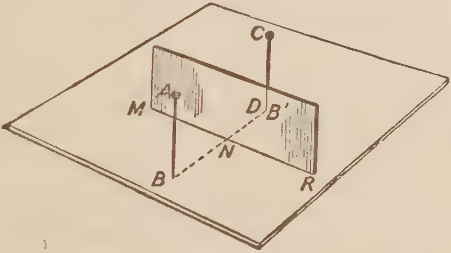


Fig. 21.

Example. *Assuming the truth of Exp. 9, show that the angle of incidence is equal to the angle of reflection.* Let M R (Fig. 22) be the mirror, B a point object, B′ its image. Any ray from B, such as B P, which impinges on M R proceeds after reflection in a direction P C as if it came from B′. Draw P Q the normal at P. The △s P N B, P N B′ are equal in all respects, for P N is common, N B = N B′ and ∠P N B = P N B′, therefore ∠ P N B = ∠P B′ N. Now since PQ is ‖ to B B′, ∠B P Q = the alternate angle P B N, and ∠C P Q = the interior and opposite angle P B′ N. Therefore ∠B P Q = ∠C P Q, or the angle of incidence is equal to the angle of reflection.

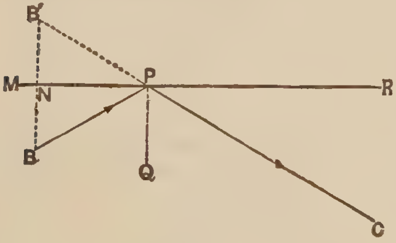


Fig. 22.

26. Reflection of a divergent pencil incident on the surface of a plane mirror. Let L (Fig. 23) denote the position of a luminous point and M M that of the mirror. Consider the reflection of any ray L A. Draw the normal A N at A; then, according to the law of reflection, the reflected ray will lie in the plane L A N, and its direction, A R, will be such that the angle R A N equals the angle L A N. Similarly, for the ray L A′, the reflected ray takes the direction A′ R′, such that R′ A′ N′ is equal to L A′ N′. Now, to an eye placed at R R′, the pencil reflected from the portion A A′ of the mirror will appear to come from a point L′ at the intersection of R A and R′ A′ (Art. 7). It can be shown that this point L′ lies on the normal to the mirror, passing through L and at the same distance behind the mirror as L is in front of it. For, through L draw the normal L O L′, and let R A produced cut it at L′. Then, by Euclid i. 29, $\angle$ L A N $=$ $\angle$ O L A,

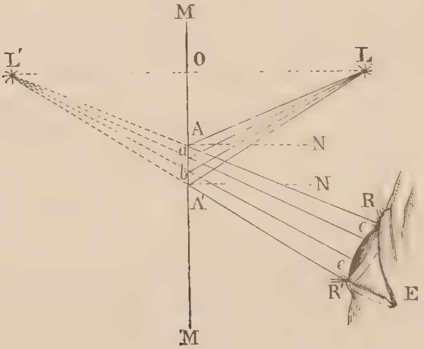


Fig. 23.

$$\text{and } \angle RAN = \angle OL'A.$$

But, in accordance with the law of reflection—

$$\angle LAN = \angle RAN;$$

$$\therefore \angle OLA = \angle OL'A.$$

∴ in the triangles A O L and A O L′ we have—
the angle A O L = the angle A O L′,
and the angle O L A = the angle O L′ A,
and the side O A common;
∴ O L = O L′ (i. 26).

Similarly, it can be shown that any other reflected ray, if produced backwards, passes through L′; and therefore, to an eye in front of the mirror, a virtual image of L is seen at L′. The image is virtual, because the rays by which it is seen do not actually come from L′, but, owing to the change of direction resulting from the reflection at the surface of the mirror, they appear to do so.

It has thus been proved, by assuming the truth of the law of reflection, that the image of a luminous point is at the same distance behind the mirror as the point itself is in front. This is the converse to the example on p. 39, where, on the assumption that the image is as far behind the mirror as the object is in front, the law of reflection was established.

26A. Reflection of a convergent pencil incident on a plane mirror. The preceding article deals with the reflection of a divergent pencil (L A A′), and shows that, after reflection, it appears to diverge from a point at the same

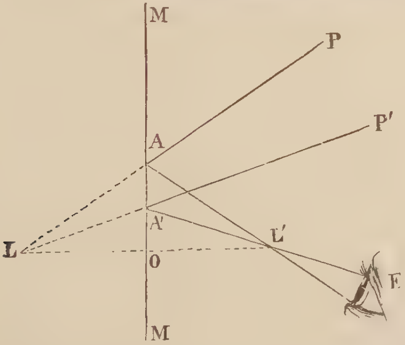


Fig. 24.

distance behind the mirror as that from which it originally diverged was in front of it. Similarly, if a convergent pencil, P L P′ (Fig. 24), converging to a point L behind the mirror, be incident at A A′, it is reflected so as to converge to a point L′, such that L O L′ is normal to the mirror, and O L = O L′. This can be proved in exactly the same way as the last case. An eye placed at E sees a *real* image at L′.

27. Image of a luminous object formed by a plane mirror. Let A B (Fig. 25) represent a luminous object placed in front of the mirror M M. As in Art. 25 the image of A is formed at A′, such that A O A′ is normal to M M and A′ O equal to A O. Similarly, the image of B is formed at B′, such that B O B′ is normal to M M and B′ O equal to B O. For all points of the object intermediate between A and B images are formed at corresponding points between A′ and B′, and thus a complete image of the object is formed at A′ B′.

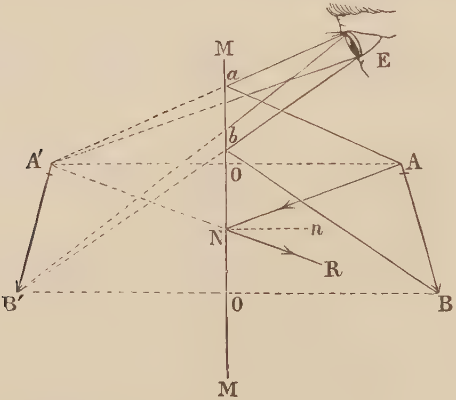


Fig. 25.

A more elaborate construction is sometimes given for determining the position of an image formed by a plane mirror, and, as the method is general and applicable to spherical mirrors, we shall briefly notice it.

It is based on the fact that the intersection of any two reflected rays determines the point on the image from which they diverge, or appear to diverge. For plane mirrors, the two rays chosen are A O (Fig. 25), incident along a normal to the mirror, and any other ray incident in any other direction, such as A N. The ray A O is reflected back along its original path, and A N is reflected along N R, making the angle of reflection R N *n* equal to the angle of incidence A N *n*, and the image of A is formed at A′, the virtual focus of the reflected rays O A and N R. Similarly, the image of any other point B is obtained, and the images of intermediate points assumed to lie on the line A′ B′, and hence A′ B′ is said to be the image of A B. When the form of the image is more complex than that considered here, the images of a number of points, sufficient to determine the complex image, must be obtained.

An eye placed at any point E (Fig. 25), in front of the mirror, sees the image A′ B′ by light reflected from the portion *a b* of the surface of the mirror, and the actual path of the *extreme* rays is shown by the lines A *a* E, B *b* E. It is evident from this that, in order that any point of an image may be seen, the line joining this point to the eye must cut the surface of the mirror, and the portion of the surface at which, by reflection, an image is seen is that portion which is intercepted by the cone having the eye at its apex and the image at its base.

Note. Different rays may cross the same place without interfering with one another.

28. Path of rays by which an image is seen. Let L′ (Fig. 23) represent the image of a luminous point L formed by the mirror M M, and imagine an eye placed at E. Draw lines joining L′ to the extremities *e e* of the aperture of the eye, and cutting the mirror at *a* and *b*; then join L to *a* and *b*, and the lines L *a e* and L *b e* define the pencil of light by which L′ is seen (cp. Art. 27). Each point of the image of a luminous object is seen in the way just described, and the extreme rays bounding the collection of pencils reaching the eye are determined in the way indicated at A *a* E, B *b* E in Fig. 25.

In connection with this question it is important to notice what must be the position of an object relative to a mirror in order that an image may be formed by that mirror. Let M M (Fig. 26) represent a mirror; then, if an object L be placed anywhere in front of the plane passing through M M, an image of that object will be formed behind this plane, at a point L′, such that L O L′ is normal to the plane and L O = L′ O. This can be proved in the same way as the proposition of Art. **26**; the figure, which corresponds to Fig. 23, shows the necessary construction, and also the path of the rays by which an eye placed at E is able to see the image L′.

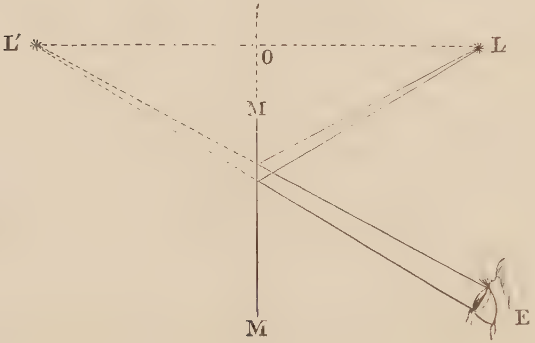


Fig. 26.

29. Lateral inversion. When the image of the face is seen in an ordinary looking-glass, we know that the image of the right eye forms the left eye of the reflected face, while the image of the left eye forms its right eye. This is a particular instance of a result of reflection known as lateral inversion. It does not affect the appearance of objects which are bi-laterally symmetrical; but with non-symmetrical objects, such as printed or written characters, the effect is sufficiently evident and well known.

Exp. 10. Write your name with a thick pen on some paper and immediately blot it. The blotting-paper shows the name backwards. Hold the blotting-paper with the name facing the mirror and view its image. The name now appears as at first written.

30. Deviation. When a ray of light is turned out of its original course it is said to suffer deviation, and the angle between its initial and final direction determines the amount of this deviation. The deviation due to a single reflection at a plane surface is easily determined. Let A O (Fig. 27) be incident on the surface M M at an angle α to the normal. Then, since the initial direction of the ray is represented by A B, and its final direction by O B′, the deviation is evidently given by the angle B O B′, which $= 180 - \text{A O B}' = 180 - 2\alpha$.

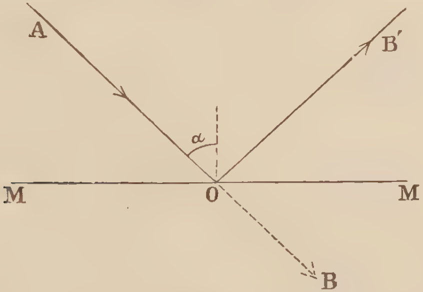


Fig. 27.

Exp. 11. Repeat Exp. 7. Now keeping the point P fixed rotate the mirror through a small angle to a new position M′ P M′. Find new positions M′ and R′ for M and R. Join up and measure the angle R P R′. Find the ratio between this angle and the angle through which the mirror has been rotated. The latter angle is exactly half the former.

31. Reflection from a rotating mirror. Let N A (Fig. 28) represent a ray incident normally on the mirror M M. If the position of the mirror remain unchanged, then N A will be reflected back along A N; but if M M be rotated, in the direction shown by the arrows, round an axis at A, into the position M′ M′, then N A will be reflected along A N′ according to the law of reflection. Now the angle $NAn = MAM'$. $\therefore NAN' = 2NAn = 2MAM'$. But N A N′ is the angle through which the reflected ray has been rotated by the rotation of the mirror through M A M′. Hence, if a mirror be turned through an angle α, the reflected ray is rotated through an angle 2α; that is, the reflected ray rotates twice as rapidly as the mirror from which it is reflected.

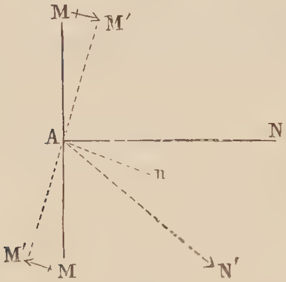


Fig. 28.

This fact finds important application in the measurement of small angular deflections. The angle is too small to be measured directly by pointer and graduated circle, hence a mirror M M (Fig. 29) is affixed to the rotating system or suspension wires, and the angle N A N′ (Figs. 28, 29) measured instead.

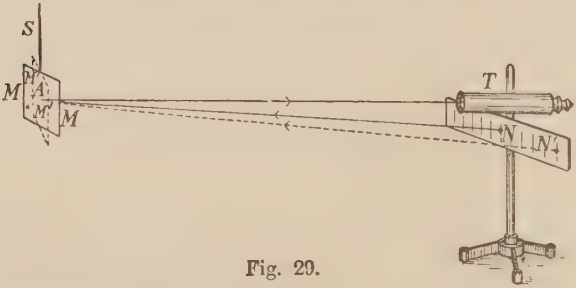


Fig. 29.

In practice a telescope and scale (Fig. 29) are employed. When the mirror is perpendicular to A N the scale-divisions at N, which is just below the telescope, are in the field of view of T. As M rotates the scale appears to travel across the field of view, and when M has reached M′ the division N′ is seen on the cross wire of the telescope.

Now $\angle NAN' = 2\angle MAM'$ (above).

$$\therefore\ MAM' = \tfrac{1}{2}\angle NAN' = \tfrac{1}{2}\tan^{-1}\frac{NN'}{AN}.$$

If the angles are small, the tangent is equal to the circular measure (see Art. 18), and therefore

$$\angle MAM' = \frac{NN'}{2AN}.$$

Since NN' and AN can be accurately measured, the angle MAM' is thus easily determined. This is sometimes called Poggendorf's, or the *Subjective*, method, and is largely used on the Continent. The most usual practice in England, however, is to use a concave spherical mirror, in which a real image of the spot of light itself is focussed on the scale (Art. 47, Figs. 54, 55). This is an *Objective* method.

32. Reflection at plane surfaces inclined to each other. Before considering particular cases of special interest, it will greatly simplify matters to notice the general principles applicable to all cases. Imagine an object A, placed between two mirrors, M_1 and M_2, inclined to each other at any angle. An image of A will be formed by each mirror; and, if the image formed by M_1 lie in front of M_2—that is, if it is anywhere in front of the plane in which this mirror lies (Art. 28)—then an image of this image will be formed by M_2. Similarly, if the image formed by M_2 lie in front of M_1, then an image of this image is formed by M_1. These are said to be images of the second order. In precisely the same way, if this second pair of images are suitably placed, a third pair (of the third order) may be formed, and so on. This multiplication of images stops when a pair is formed in the space behind both mirrors—that is, within the angle vertically opposite to that in which the object is placed. We shall now consider a few special cases.

Exp. 12. Take two plane mirrors; stand them vertically on a table with two upright sides in contact. If the mirrors are already hinged as in Fig. 30 so much the better. Place a candle between them and, starting with the mirrors in the same plane, gradually swing them round the common sides until they close in upon the candle.

Note the formation of images. When the angle between the mirrors is 90° there are 3 images, when 60° 5 images, when 45° 7 images, etc., all arranged on a circle whose centre is on the common side and which passes through the candle. Finally, place the mirrors parallel to each other: the number of images is now infinite.

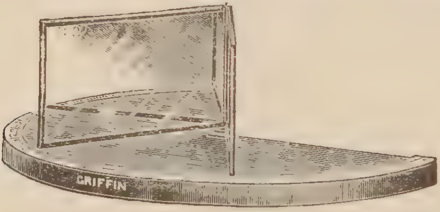


Fig. 30.

(1) **Parallel mirrors.** Let M_1 and M_2 (Fig. 31) represent two parallel mirrors, and A an object placed between them. It is evident that since the mirrors are parallel no image can be formed behind both, and hence every pair of images gives rise to another pair, and thus an infinite series of images may, theoretically, be formed. Through A draw $N_1 N_2$ normal to both the mirrors, and produce it indefinitely on both sides. In obedience to the law of reflection all the images must lie on this line, and their positions on it will depend on the position of A between M_1 and M_2, and on the distance between these mirrors. Consider first the reflection from M_1; an image of A is formed at A_1' on the normal through A, and so placed that $A_1' N_1$ equals $A N_1$. Similarly an image of A_1' is formed by M_2 at A_1'' in such a position that $A_1'' N_2 = A_1' N_2$; A_1'' in turn gives rise to A_1''' by reflection at M_1, and so on. In the same way, beginning with the first reflection at M_2, the images A_2', A_2'', etc., are formed by successive reflection at M_2 and M_1. In Fig. 31 the positions of the images up to the third order are shown, and, to distinguish them, the suffix attached to A denotes the mirror at which the *first* reflection took place, and the dashes indicate the order of the image. Thus the

series A_1', A_1'', A_1''' . . . is formed by successive reflections from M_1 and M_2, beginning with reflection at M_1; similarly the series A_2', A_2'', A_2''' . . . is formed by successive reflections from M_2 and M_1, beginning with M_2. The members of each series are so related that any one may be considered as the image of the one immediately preceding it: for example, A_1''' may be considered as the image of A_1'' formed by the mirror M_1, and consequently $A_1''' N_1$ equals $A_1'' N_1$.

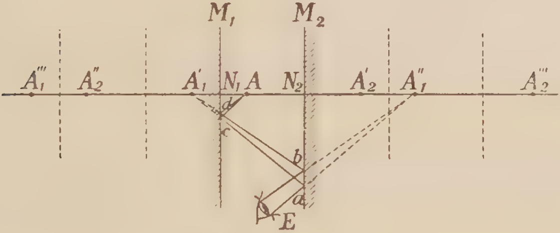


Fig. 31.

To determine the path of rays by which any image is seen, the following construction should be employed. Let it be required to find the path of the rays by which an eye at E sees the image A_1''. First trace this image back to A; A_1'' is an image of A_1', which is itself an image of A. Now join the extremities of the aperture of the eye to A_1'' by lines cutting M_2 at a and b, and mark the real parts of this path, which, since the rays cannot penetrate the mirror, must lie between the eye and $a\,b$. Next join a and b to A_1' by lines cutting M_1 in c and d, and mark $a\,c$, $b\,d$ as the real portions of this path. Then finally join c and d to A, and the twice reflected pencil passing from A to E indicates the required path. From this it is evident that an image of the *second* order, A_1'', is seen by *two* reflections, and similarly an image of the n^{th} order would be seen by n reflections. The mirror from which the last reflection takes place—that is, the mirror in which the image is seen—depends upon whether n is odd or even. In either series of images the *odd** members are seen by reflection from the mirror at which the *first* reflection takes place, while the even members are seen in the other mirror. At each reflection there is some loss of light, depending in amount on the polish of the reflecting surface; and, as a consequence, the higher the order of any image the fainter it appears, until finally it becomes too faint to be visible.

(2) **Mirrors inclined at right angles.** Let O M_1 and O M_2 (Fig. 32) represent two mirrors at right angles to each other, and A an object placed between them. Then an image A_1' is formed by O M_1 and A_2' by O M_2. Then A_1' lies in front of O M_2, and therefore an image A_1'' is formed by that mirror. Also, A_2' is in front of O M_1, and therefore gives rise to the image A_2'', which from the geometry of the figure is evidently coincident with A_1''. An eye placed anywhere within the proper limits (Art. 27) sees three images, A_1',

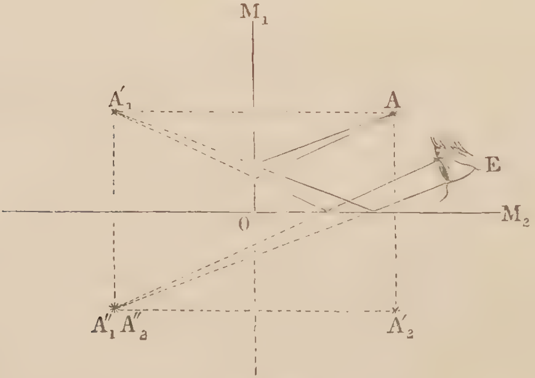


Fig. 32.

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* That is, the 1st, 3rd, 5th, etc.

A_1'' *or* A_2'', and A_2', at the three corners of the rectangle $A_1' A_2'$. *Both* the images A_1'' and A_2'' cannot be seen at the same time; an eye placed within the angle $M_1 O A$ sees the image A_2''; while one placed in $M_2 O A$ sees A_1''. The figure shows the path of the rays by which the image A_1'' may be seen by an eye placed at E. The method of determining this path is indicated by the dotted lines, and is similar to that explained above for parallel mirrors. The actual path of the rays necessarily lies within the angle $M_1 O M_2$.

(3) **Mirrors inclined at any angle.** Let $O M_1$ and $O M_2$ (Fig. 33) represent two mirrors inclined at the angle $M_1 O M_2$ and A an object placed between them. With O as centre, describe a circle passing through A and cutting $O M_1$ and $O M_2$ in N_1 and N_2 respectively. Then all the images of A must lie in the circumference of this circle. For consider the image A_1'; according to the law of reflection it is so

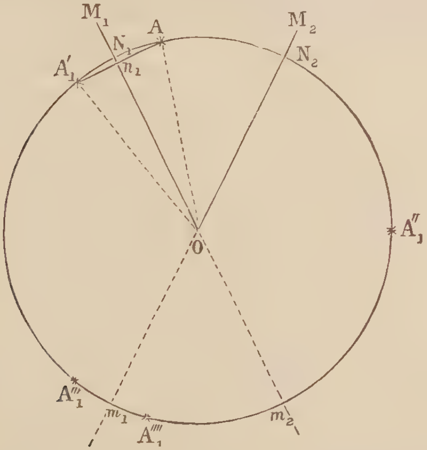

Fig. 33.

placed that $A n_1$ equals $A_1' n_1$, and $A A_1'$ is at right angles to $O M_1$. Hence, in the triangles $O A n_1$ and $O A_1' n_1$ we have $A n_1$ equal to $A_1' n_1$, $n_1 O$ common, and the angle $A_1' n_1 O$ equal to the angle $A n_1 O$; therefore $O A_1'$ equals $O A$ (Euc. i. 4), and A_1' lies on the circumference of the circle passing through A. Similarly for any other image.

The successive formation of images is exactly similar to that described for parallel mirrors; each mirror gives rise to a separate series of images, but in this case the number of images that can be formed is limited, the last member of each series being that formed within the angle $m_1 O m_2$. Fig. 33 shows the positions of the members of the series formed by first reflection from M_1. In general, the last members of the series, formed on the arc $m_1 m_2$, have different positions; but when the angle $M_1 O M_2$ is an aliquot part of 360°, then these images are coincident.

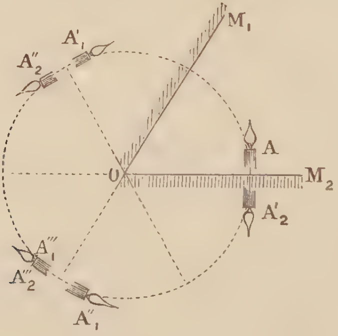

Fig. 34.

Figs. 32 and 34 illustrate this; in Fig. 32, $M_1 O M_2$ (90°) is one-fourth of 360°, and the images A_1'' and A_2'' are coincident; similarly, in Fig. 34, $M_1 O M_2$ (60°) is one-sixth of 360°, and the images A_1''' and A_2''' are coincident. From Fig. 34 it will be seen that the number of images formed is 5 when $M_1 O M_2$ is *one-sixth* of 360°—that is, if the angle $M_1 O M_2$ be $1/n^{th}$ of 360°, then the number of images formed is $(n-1)$. Or, if $M_1 O M_2$ be denoted by θ, then, when θ is an aliquot part of 2π, the number of images formed is given by $\left(\frac{2\pi}{\theta}-1\right)$.

Exp. 13. Place two mirrors, OM_1, OM_2 (Fig. 35), on a sheet of cartridge paper at an angle of 60° with each other, and between them press a pin A into the paper so that it stands upright. Looking into the mirrors a series of images A_1', A_2'', A_2''' (A_1'''), A_1'', A_2' will be seen. Locate the positions of these images by other pins as in Exp. 9. Now remove pins and mirrors and prove that (1) the images all lie on a circle whose centre is O and radius OA, (2) the angles AOA_1', $A_2''OA_2'''$, $A_2'OA_1''$ are equal to one another, and (3) the angles AOA_2', $A_1''OA_2'''$, $A_2''OA_1'$ are also equal to one another. Show also that if an eye be placed at E, the path of a ray of light apparently coming from the image A_2''' is ABCDE. The construction is obvious. (Join the eye E to the image viewed, in this case A_2'''. This line cuts the mirror OM_2 in D. A_2''' is the image of A_2'' in OM_2. Therefore join D to A_2'', cutting OM_1 in C. A_2'' is the image of A_2' in OM_1. Therefore join CA'_2, cutting OM_2 in B. A_2' is the image of A in OM_2. Therefore join B to A.)

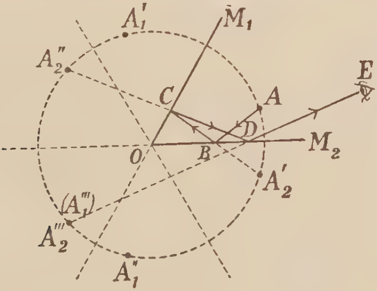

Fig. 35.

The symmetrical distribution of images obtained by repeated reflection between two mirrors when they are inclined at an angle which is an exact submultiple of 180° is the principle of the **kaleidoscope**. Two long narrow mirrors, inclined to each other at an angle of 60°, are placed in a slightly longer tube. One end of the tube is closed by a metal disc, pierced at the centre, with a hole through which the observer looks; at the other end a plate of clear glass fits into the tube close up to the mirrors; and a short distance beyond it, at the end of the tube, is a similar plate of ground glass. Between these two glass plates little pieces of coloured glass, etc., are loosely placed, and, with their images, form beautiful and symmetrical patterns visible to an eye placed at the other end of the tube. On rotating the tube the pieces of glass change position, and thus the pattern seen is continually changing. Sometimes three mirrors are employed, the arrangement being such that the cross section of the three is an equilateral triangle. Each pair of plates acts in the way described above, so that the arrangement gives rise to intricate but symmetrical patterns, which are capable of giving material aid to designers.

33. Irregular or Diffuse Reflection. When a parallel beam from a magic lantern in a dark room falls on a piece of white card, the light after incidence is not confined to one course, but is scattered in all directions. From anywhere in front the card is brightly visible, and the room is no longer wholly dark. If a mirror had been employed, practically all the light would have been reflected in some definite direction. At first sight there seems to be a great difference between the two phenomena; and it has been sought to explain the behaviour of the card by comparing it to a mirror with many small facets, which reflect the light quite regularly, but in different directions, because they themselves are at different slopes. This phenomenon does in fact occur when light is refracted from water on which are ripples, and accounts for the wide luminous path seen on a lake under the sun and moon; but it is not just what takes place when light is diffused by a card. The fact is that diffusion, and not reflection, is the fundamental phenomenon. Diffusion cannot therefore be explained by reflection; but the latter is a consequence of the former. A full explanation of this would be impossible; but the essential fact is that light consists of a series of waves. Waves striking any obstacle are always scattered in all directions; but this scattering produces a regular reflected wave whenever the obstacles are ranged in an even surface whose inequalities are not large compared to the wave-length. Thus the sound waves from the tick of a watch are about an inch long; they are regularly reflected by a surface whose inequalities are less than, say, half an inch high. Water waves on the sea may be over a hundred feet long; they are reflected easily by a somewhat irregular coast line. Sound waves, six feet long, are reflected so as to produce a true echo from a hedgerow. Light waves have a length of 40 to 75 millionths of a centimetre (one to two millionths of a foot), and are well reflected from polished metal or glass, or the surface of a liquid in which the inequalities are of this order of magnitude, but badly reflected from cardboard, in which they are larger. By means of the cardboard we can, however, prove another consequence of the wave theory, that a ray is reflected fairly regularly if it be very oblique indeed, so as to be almost parallel to the surface, even if the surface be rough (see Art. 34).

Twilight is explicable by diffusion. Clouds, dust, and other floating particles in the atmosphere are illuminated by the sun some time after it has set at any particular place. These scatter the light in all directions, some of the scattered rays of course reaching the earth, illuminating it for some time after sunset. Moreover, some of the scattered light is transmitted to other particles of the atmosphere farther away from the sun, and these reflect the rays a second time ; the result of these second reflections is to still further increase the duration of twilight. Twilight is said to end when this scattered light becomes imperceptible. By observation this has been found to occur when the sun is at a depth of about 18° below the horizon.

If the earth had no atmosphere, surfaces on it exposed to the sun's rays would be dazzlingly bright, whilst all other surfaces would be in black shadow, except such parts as might be illuminated by reflection and diffusion from surrounding surfaces. This state of things indeed exists on the moon, where the contrast between light and shade and the sharpness of shadows are extremely great.

It is only by means of the light they scatter that all bodies, except self-luminous ones, are made visible to us. The scattering of the light of the sun by white clouds is the cause of the difference between ordinary daylight with its soft gradations of light and shades, and direct sunlight with its intense lights and deep shadows. For the effect of scattering on the colour of the light see **Art. 108**

34. Plane Mirrors. It is extremely difficult to make accurate plane surfaces. The simplest test of flatness that can be applied is that of reflection.

Exp. 14. Make a smooth round hole in a piece of tinfoil and place it in front of a bright light, L (Fig. 36). Fix a telescope, T, in a stand at a distance from the foil and focus the telescope on the hole. On the bench midway between the telescope and lamp lay the surface, S, to be tested, and incline the telescope until the hole is seen by reflection from the surface.

If the surface is plane the image will still be sharp, if irregular the image will be ill-defined and may spread out into a large and irregular blur, like the image of the moon on a lake. Test various kinds of glass, from ordinary window glass to optically plane glass.

The experiment may be varied by holding the surface horizontally in the hand, just below the level of the eye, and viewing in it the images formed by reflection of the bars of a well-lighted window frame. If the bars appear sharp and straight the surface is plane, but if wavy and crumpled the surface is irregular.

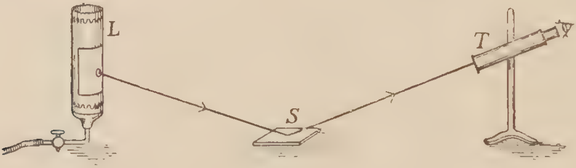

Fig. 36.

The proportion of the incident light reflected at a surface depends very much on the nature of the bodies in contact, on the state of polish of the surface and the angle of incidence. It increases with the angle of incidence.

35. Principle of Least Time. The path which light takes to travel from one point to another by reflection at one or more mirrors is always such that the time taken is the least possible.

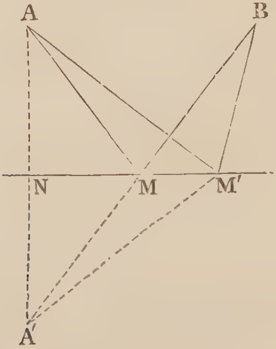

Fig. 37.

Example. A ray of light starts from A (Fig. 37), meets a plane reflecting surface at M, and is reflected to B. Prove that A M B is the shortest possible path from A to B by way of the mirror.

If A M B be not the shortest path, let *any* other path A M′ B be shorter. Draw A N A′ normal to the mirror, and produce B M to meet A A′ in A′.

Then, since $AN = A'N$ we have, by Euclid i. 4, $AM = A'M$ and $AM' = A'M'$.

But $A'M' + M'B > A'B > A'M + MB$ (Euc. i. 20).

$\therefore\ AM' + M'B > AM + MB.$

EXERCISES III.

1. Given a knitting needle with a little soft wax on one end, adjust it so as to be perpendicular to the surface of a mirror.

2. Prove that, when a ray of light is reflected at a plane mirror, the angle of incidence is equal to the angle of reflection.

3. Given a piece of cardboard on which are ruled two straight lines which meet at a point, a small mirror which can stand with its reflecting surface at right angles to the cardboard, some pins, and a ruler, draw a line to bisect the angle between the two given lines on the card.

4. Place a candle between two mirrors hinged together, and prove experimentally that the number of images visible when the angle between the mirrors is $\theta°$ is equal to $\frac{360}{\theta} - 1$. Take, as values of θ, 90°, 72°, 60°, 45°, 40°, 36°, 30°. What happens when $\theta = 0°$, *i.e.* when the two mirrors are parallel to each other?

5. Smoke the outside of a glass tube. Cover one end with tinfoil and prick a pinhole in the centre of the tinfoil. Look through the other end at a candle. Explain the formation of the concentric circles of light.

6. Define the *angle of incidence*, *angle of reflection*, *plane of incidence*, and *plane of reflection*. State the Laws of Reflection of light, and describe some form of apparatus by which they may be demonstrated.

7. Draw a diagram showing the formation of the image of a luminous point in a plane mirror.

8. When a horizontal beam of light falls on a vertical plane mirror which revolves about a vertical axis in its plane, show that the reflected beam revolves at twice the rate of the mirror.

9. Explain what is meant by *lateral inversion*, and draw a diagram to illustrate your answer.

10. A plane mirror which is first 1 foot from an object is then moved back 1 foot *parallel to itself*. How far does the image move? Give a diagram in illustration.

11. A source of light is seen by reflection in two vertical plane mirrors placed against the two walls in a corner of a square room. Construct a figure showing exactly the path of a beam which enters the eye of an observer after two reflections, one at each mirror.

12. A small object is placed between two parallel mirrors as in Fig. 31. The distance between the mirrors is 6 inches, and the object is placed 2 inches from one of them. Find the distances between the corresponding members of the two series of images formed; also the distances between the odd members of each series, and between the even members of each series.

13. The sun is 30° above the horizon, and its image is observed in a tranquil pool. What, in this case, is the angle of incidence and reflection?

14. A man, 6 feet high, sees his image in a plane mirror hung vertically. The top of the mirror being 6 feet from the ground, determine its smallest length in order that the man may see his full-length image in it.

15. Find the deviation produced by reflection at a plane mirror, when the angle between the incident and reflected rays is 80°.

16. Show, that if a ray of light be incident at any angle, on one of two mirrors inclined at right angles to each other, then the ray is reflected from the second mirror in a direction parallel to its original direction.

17. A mirror revolves about a horizontal axis parallel to its surface. Show how to find if the reflecting surface is accurately parallel to the axis of revolution.

18. A mirror, scale, and telescope are used to measure the deflection of a suspended system. The scale is distant one metre from the mirror, and during the movement of the mirror the scale reading alters from 14 cms. to 44 cms. Find approximately the angle of deflection of the system.

19. Make a measured drawing showing the positions of all the images formed by two mirrors inclined to each other at 45°, of an object placed between the mirrors.

20. Two mirrors, M_1 and M_2, are inclined to each other at 50°, and an object is placed between them. Make a measured drawing showing in black the situation of all the images formed where the rays from the object strike M_1 first, and in red the situation of all the images where the rays first strike M_2.

21. An object is placed $\frac{1}{2}$ in. from one plane mirror and 1 in. from another plane mirror parallel to the first, so that the object is between them. Make a measured drawing showing the positions of all the images up to the fourth order.

22. A horizontal narrow strip of plane mirror is hung up against the wall of a room on a level with the eye of the observer. Draw a diagram showing how much of a side wall of the room will be visible by reflection from the mirror.

CHAPTER V.

REFLECTION AT SPHERICAL SURFACES.

36. Preliminary definitions. Mirrors of spherical, parabolic, and cylindrical curvature are used in optical instruments. We shall in this chapter confine the discussion to spherical mirrors.

A spherical mirror, A A′ (Fig. 38), is usually a very small segment of a spherical surface, and may be either

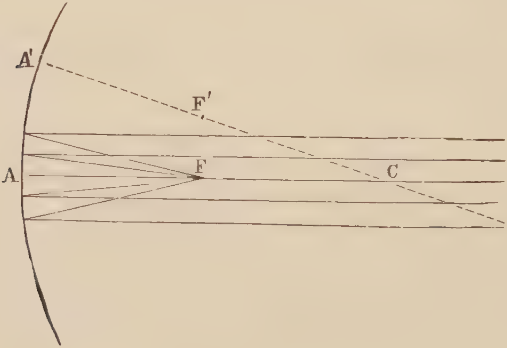

Fig. 38 (A).

concave (Fig. A) or convex (Fig. B). The centre, C, of the spherical surface of which the mirror is a part is called the *centre of curvature* of the mirror. The line C A, joining the centre of curvature and the central point, A, of the mirror, is the *principal axis* of the mirror, the point A being sometimes called the *pole* or *centre of the face.* Any other line C A′ drawn through C and cutting the mirror is called a *secondary axis*, and is, like the principal axis, a normal to the mirror. A section of the mirror by a plane passing through the pole and the centre of curvature is called a *principal section*.

The *aperture* of the mirror is the angle enclosed by two straight lines drawn from the centre of curvature to opposite points in the edge of the mirror. We shall at first limit the discussion to mirrors of small aperture—say not exceeding 10°, though for the sake of clearness the diagrams will show greater apertures.

When a parallel pencil of light is incident on a spherical mirror, in a direction parallel to the principal axis, the reflected pencil converges to or diverges from a point F on the principal axis. This point is called the *principal focus* of the mirror, and the distance, A F, between the principal focus and the pole of the mirror is termed the *focal length* of the mirror. If the pencil is incident parallel to a secondary axis, the reflected rays are, in a similar way, brought to a focus at a point F′ on that axis. In the case of spherical mirrors, it is not strictly true to say that the reflected pencils meet accurately at a point; if the pencil is small, this is approximately the case, but with large pencils the outer rays are reflected to points nearer the mirror than the others. This irregularity of reflection from a spherical surface is called *spherical aberration* (see Art. 43).

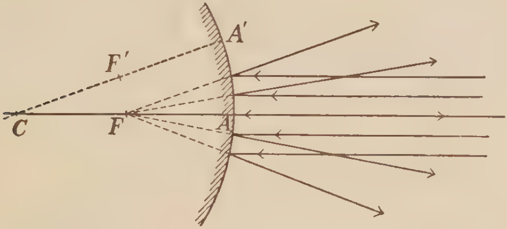

Fig. 38 (B).

Exp. 15. Take a concave mirror (*i.e.* a hollow mirror) and look into it at the reflection of a candle placed some distance away: note that the image is inverted. Hold the candle up close to the mirror: the image is erect. Remove the candle gradually from the mirror: at a certain distance the image becomes indistinct and vanishes; beyond that the image is inverted.

Exp. 16. Take a convex mirror (*i.e.* a bulging mirror) and hold a candle in front of it and at various distances from it: note that the image of the candle is always upright.

Exp. 17. Hold a curved surface near the eye and look along it towards a bright light. Place a straight edge across and close to the surface, observe its image: it is curved. If the surface is concave the middle part of the image is bent away from the straight edge; if convex, the ends are bent away.

37. Construction for reflected ray. Let P Q be any ray incident at Q on a spherical mirror [concave Fig. 39 (A), or convex Fig. 39 (B)]. At Q draw the normal, Q N, to the reflecting surface, by joining C Q and producing it if necessary. Then, in accordance with the laws of reflection,

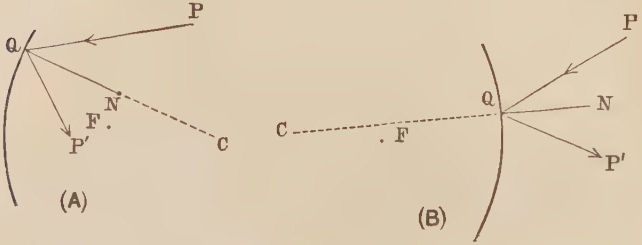

Fig. 39.

the reflected ray Q P′ is obtained by drawing it in such a direction that the angle of reflection P′ Q N is equal to the angle of incidence P Q N.

From this it is evident that a ray incident along a normal is reflected back along the path by which it came. Also, from Art. 36, a ray incident parallel to the principal axis is reflected through the principal focus. These two particular cases of reflection should be carefully remembered.

38. Position of principal focus. Let P Q (Fig. 40) be a ray incident on the concave mirror A Q, in a direction parallel to the principal axis C A. Then P Q is reflected through the principal focus F, and the angle P Q C is equal to the angle F Q C. But the angle P Q C is equal to the angle F C Q (Euc. i. 29); therefore F Q C = F C Q, and therefore F Q = F C.

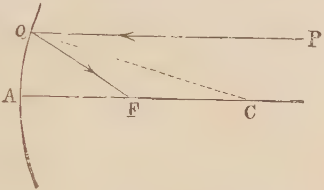

Fig. 40.

Now, if A Q is small, F Q is approximately equal to F A, and therefore F C equals F A (approximately). That is, the principal focus F is midway between the pole A and the centre of curvature C; and, if A F be denoted by f and A C by r, we have $f = \frac{r}{2}$; or *the focal length of a spherical mirror, for rays incident on a small portion of its surface near the pole, is equal to half the radius of curvature of that mirror.*

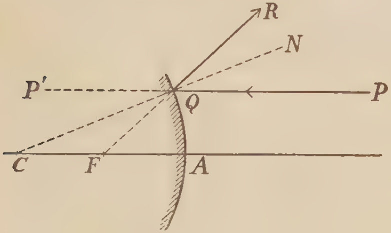

Fig. 41.

An exactly similar proof is applicable in the case of a convex mirror. The principal focus F (Fig. 41) is in this case on the backward prolongation of the reflected ray, and is, of course, *virtual.*

Note. (1) If the vertical angle of a triangle is bisected *internally*, the bisector divides the base *internally* into segments which are in the same ratio as the other sides of the triangle adjacent to these segments (Euclid vi. 3).

(2) If the vertical angle of a triangle is bisected *externally*, the bisector divides the base *externally* into segments which are in the same ratio as the other sides of the triangle adjacent to these segments (Euclid vi. A).

39. Conjugate foci. Let P (Fig. 42) represent the position of a luminous point on the principal axis of the concave mirror A Q. Then, Art. **27**, the image of P will be formed at the intersection, after reflection, of any two rays coming from P. Consider the rays P A incident along the normal to the mirror, and P Q incident at Q; P A is reflected back along A P and P Q is reflected along Q P′, making the angle of reflection P′ Q C equal to the angle of incidence P Q C. Let the reflected rays A P and Q P′ intersect at P′; then P′ is the image of P, and lies on the principal axis of the mirror. Also, since P Q C = P′ Q C, then—

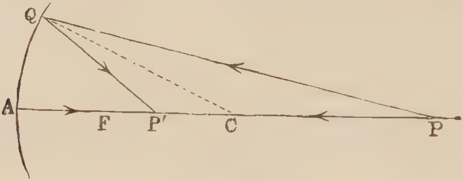

Fig. 42.

$$\frac{QP'}{QP} = \frac{P'C}{CP}. \qquad \text{(Euclid vi. 3.)}$$

But, if A Q is small, then P Q = P A, and P′ Q = P′ A, and therefore—

$$\frac{AP'}{AP} = \frac{P'C}{CP}$$

If now A C be denoted by r, A P by u, and A P′ by v, then P′C = AC − AP′ = $r - v$, and C P = AP − AC = $u - r$. And the above proportion becomes—

$$\frac{v}{u} = \frac{r-v}{u-r}.$$

$$\therefore\ u(r-v) = v(u-r).$$

$$\therefore\ ur + vr = 2uv.$$

Dividing by urv, then—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

But, by Art. 38—

$$f = \frac{r}{2}. \quad \therefore \frac{2}{r} = \frac{1}{f}, \text{ and}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \qquad \text{(1.)}$$

where u denotes the distance of the luminous point P from the pole of the mirror, v denotes the distance of the image of P from the pole of the mirror, r denotes the radius of curvature of the mirror, and $f\left(=\frac{r}{2}\right)$ denotes the focal length of the mirror.

This may be expressed in words by saying that *the algebraical sum of the reciprocals of the distances of the luminous point and its image from the pole equals the reciprocal of the distance of the principal focus from the pole.*

The relation thus obtained is of great importance. It will be noticed that, in the case here considered, all the distances involved are measured in the same direction from A. When this is not the case, it is necessary to adopt some convention as to sign. The most general convention, and the one adopted throughout this book, is *to consider all distances measured in a direction* **opposed** *to the incident light as* **positive**, *and distances measured in the* **same** *direction as the incident light as* **negative**. With this convention the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ is applicable to all cases of reflection at spherical mirrors.

The points P and P′ connected by this relation are said to be *conjugate foci*, because of the fact that either point may be considered as the image of the other. From the construction it is evident that the image of a luminous point at P′ would be formed at P, just in the same way as the image of P is formed at P′. This may be illustrated experimentally by means of a candle and a concave mirror. If the flame of the candle be placed, at any point beyond C, on the principal axis of the mirror, an image of the flame will be seen between C and the mirror. The position of this image can be marked by adjusting the position of a needle until it appears to coincide with the image. It will then be found that if the candle flame be placed at the point marked by the needle, the image will be seen at the point originally occupied by the flame.* If the luminous point P is not on the principal axis, then its conjugate focus, P′, will be on the secondary axis passing through P, and, distances being measured along the axis, the relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, can be established in the way explained above In fact, the two cases are identical, for the geometrical relations of a secondary axis to a spherical mirror are exactly the same as those of the principal axis.

The same relation for conjugate foci can also be proved for a convex reflecting surface. Let P (Fig. 43) represent

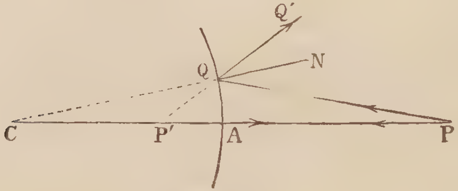

Fig. 43.

a luminous point placed in front of the convex mirror A Q. Then, as in the case of the concave mirror, an image of P is formed at P′ on the axis passing through P. In this case P′ is a *virtual focus*, from which the reflected rays A P and Q Q′ appear to diverge. To determine the position of P′ we have—

$$\frac{QP'}{QP}=\frac{P'C}{PC}. \qquad \text{(Euclid vi. A.)}$$

Also, if A Q be small, this proportion becomes—

$$\frac{AP'}{AP}=\frac{P'C}{PC}=\frac{AC-AP'}{PA+AC}.$$

In this equation, A C, A P, A P′ bear positive numerical values. Now, using the same meanings as above for u, v

---

* This is an instance of the reversibility of light (Art. 22).

and r, it is evident that A C must be replaced by $-r$, A P by u, and A P′ by $-v$. Therefore, substituting, we get—

$$\frac{-v}{u} = \frac{-r-(-v)}{u-r} = -\frac{r-v}{u-r}.$$

Cross-multiplying, this becomes—

$$vu - vr = ur - uv.$$

$$\therefore ur + vr = 2uv.$$

and dividing through by urv, we get—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

That is—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

In an exactly similar way this formula may be established for the reflection of a convergent pencil at a spherical surface, and the reader will find it an instructive exercise to draw the necessary figures, and deduce the formula therefrom.

In proving theorems such as the above in which some quantities may be positive and some negative, students are recommended to take a case in which all the quantities are positive. If this be done, confusion arising from questions of signs will not occur.

Examples. In working out examples on mirrors the student should pay attention to the following points:—

1. **The sign convention.** See above.

This convention applies to all cases, wherever the distance considered may be measured from. In applying the above formulae the following points must be noticed :—

2. *On substituting a numerical value for any of the symbols, the sign of the former must always be attached.*

For example, if in formula (1), $u = 6$ and $v = -8$, then, on substitution we get—

$$\frac{1}{-8} + \frac{1}{6} = \frac{1}{f} = \frac{2}{r}.$$

$$\therefore \frac{1}{24} = \frac{1}{f}$$

$$\therefore f = 24 \text{ and } r = 48.$$

3. *In applying a formula to determine one of the involved distances, the others being known, no sign must be given to the unknown distance.* Thus, in the above example, no sign is at first given to f; but the

result, when worked out, shows it to be positive—that is, the mirror is concave.

4. When distances are measured from the pole of the mirror [as in formula 1], the radius of curvature and focal length are *positive* for a *concave* mirror, and *negative* for a *convex* mirror. This is in accordance with the sign convention given above, and needs special notice only as a reminder.

5. *Always draw a fairly accurate figure* representing given conditions. This prevents mistakes as to sign.

(1) *An object is placed* 15 *cms. in front of a concave mirror of* 30 *cms. focal length. Find the position of the image. Is the image real or virtual?*

Here we have given us—

$$u = 15;\ f = 30.$$

Hence, substituting in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

we have— $$\frac{1}{v} + \frac{1}{15} = \frac{1}{30}. \quad \therefore \frac{1}{v} = -\frac{1}{30}.$$

That is, the image is 30 cms. *behind* the mirror, and is therefore *virtual.*

(2) *A pencil of rays, converging to a point 20 cms. behind a mirror, is brought to focus, by reflection from its surface, at a point 10 cms. in front of the mirror. Determine whether the mirror is convex or concave, and find its radius of curvature.*

$$\text{Here } u = -20,\ v = 10.$$

$$\therefore \frac{1}{10} - \frac{1}{20} = \frac{2}{r}. \quad \therefore \frac{2}{r} = \frac{1}{20},$$

$$\text{or } r = 40.$$

That is, the mirror is concave, and its radius of curvature is 40 cms.

(3) *An object, 3 cms. in length, is placed 20 cms. in front of a convex mirror of 12 cms. focal length. Find the nature and position of the image.*

$$\text{Here } u = 20,\ f = -12.$$

$$\therefore \frac{1}{v} + \frac{1}{20} = -\frac{1}{12}.$$

$$\therefore \frac{1}{v} = -\frac{1}{20} - \frac{1}{12} = -\frac{2}{15}.$$

$$\therefore v = -7{\cdot}5.$$

That is, the image is 7·5 cms. *behind* the mirror, and is therefore *virtual.*

Note. At this stage the student should become familiar with the meaning of the terms infinite and infinity. A quantity becomes *infinite* when its value becomes greater than any value we can assign to it. If the value of any quantity q is infinite, this is expressed by writing $q = \infty$.

The term *infinity* will be best understood from its use in the statement that parallel straight lines meet at infinity. If any straight line O A be produced to A′, in the direction O A, until it is of infinite length, the point A′ will be at infinity.

Consider the ratio $\frac{a}{x}$. If x becomes infinite, the ratio becomes $\frac{a}{\infty}$, and the value of this expression for finite values of a is zero. That is

$$\frac{a}{\infty} = 0$$

where a is *any* finite quantity.

For example suppose a is 6. Give x values of 100, 10000, 1000000, then $\frac{a}{x}$ is $\frac{6}{100}, \frac{6}{10000}, \frac{6}{1000000}$, *i.e.* ·06, ·0006, ·000006. If now x is made infinitely great $\frac{a}{x}$ = ·0000... to an infinite number of ciphers, which is practically zero.

40. Relative position of conjugate foci. In the preceding article it has been shown that the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ holds good for all cases of reflection at a spherical surface. By a general discussion of this formula it is possible to determine the position of P′ for any given position of P. For example, if u be infinite—that is, if the incident light be parallel—then we have—

$$\frac{1}{v} + \frac{1}{\infty} = \frac{2}{r}.$$

Therefore, since $\frac{1}{\infty} = 0$,

$$\frac{1}{v} = \frac{2}{r} \text{ or } v = \frac{r}{2}.$$

This means that if a pencil of parallel light be reflected at the spherical surface, its focus, after reflection, is on the axis parallel to the incident light at a point whose distance from the mirror is equal to half the radius of curvature of the mirror (Art. 38).

In addition to the above, the following general rule will be found of great use in determining the motion of the image corresponding to any given motion of the object along an axis of the mirror:—*When an image is formed by reflection, any motion of the object, in a given direction along an axis of the mirror, causes motion of the image in an* **opposite** *direction along the same axis.*[*]

By the application of these rules we may trace the position of P, as P travels from infinity up to a spherical mirror. Thus, considering first a **concave** mirror, when the incident light is parallel—that is, when P is at infinity in front of the mirror—P′ is at F (Fig. 44). As P travels from infinity up towards C, P′ travels, *in the opposite direction*, from F towards C. At C, P and P′ meet, and image and object coincide. As P travels from C to F, P′ travels, in the opposite direction, from C to infinity, and the initial relations of P and P′ are now reversed. As P further travels from F to A, P′ travels, *in the opposite direction*, from infinity to A—that is, P′, after disappearing at infinity *in front* of the mirror, reappears from infinity *behind* the mirror and travels up to A.

The case of a **convex** mirror is simpler. The principal focus, F (Fig. 44), is *behind* the mirror; hence, as P travels from infinity up to the mirror, P′ travels, in the opposite direction, from F up to the mirror.

Now, if P be a real luminous point, or small object, such as a candle flame, it can evidently travel no further than A, and thus we have traced all possible positions of the image of a real luminous point placed anywhere in front of a spherical mirror.

It may be useful to summarise the points of practical importance mentioned in this article.

I. Concave mirror.

1. Luminous point between $+\infty$ and C. Real image between F and C.

2. Luminous point at C. Real image at C.

3. Luminous point between C and F. Real image between C and $+\infty$.

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[*] In the case of a plane mirror, any *normal* to its surface may be considered as an axis.

4. Luminous point between F and A. Virtual image between A and $-\infty$.

5. Luminous point at A. Image at A.

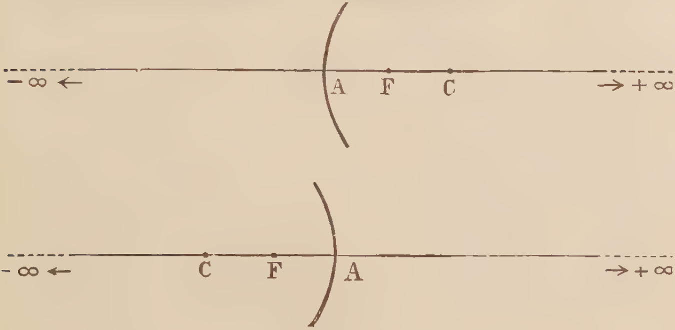

Fig. 44.

II. **Convex mirror.**

1. Luminous point between A and $+\infty$. Virtual image between A and F.

Example. Draw a curve showing the relation between the distance of a point and that of its image as measured from a concave mirror as the distance of the point is progressively varied. Art. 40 gives us the relative distances, and it is best to begin by putting these in tabular form thus:—

u	v
∞	f
$< \infty$ but $> r$	$> f$ but $< r$
r	r
$< r$ but $> f$	$> r$ but $< \infty$
f	$\pm \infty$
$< f$ but $> \frac{f}{2}$	$> -\infty$ but $< -f$
$\frac{f}{2}$	$-f$
0	0
$-f$	$\frac{f}{2}$
$-\infty$	f

Plotting these on squared paper we obtain the following curves. The curves A B and C D form a rectangular hyperbola.

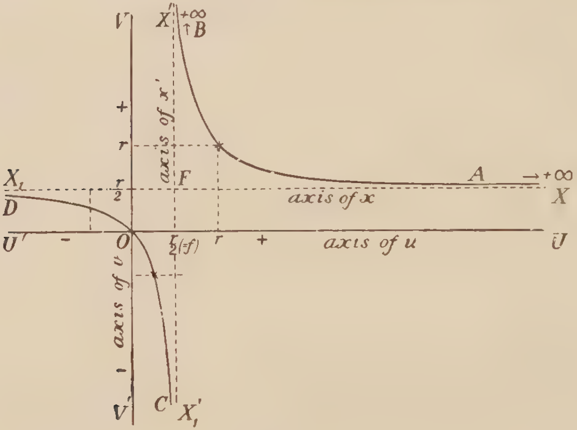

Fig. 45.

Note that a curve which gradually approaches a dotted line goes to infinity on one side of the line, and reappears at an infinite distance in the opposite direction on the other side of the line.

41. Formation of images by spherical mirrors. When a luminous object is placed in front of a spherical mirror an image is formed, which may be *real* or *virtual* according

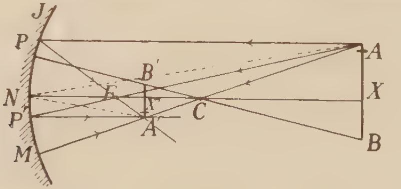

Fig. 46.

to the circumstances of the case. If real, the image is formed in front of the mirror, and can be received on a screen; but if virtual, it appears to be behind the mirror, and cannot be received upon a screen. It may, however, be located by means of a pin as in Exp. 9 (see Exp. 22). The paths of the rays by which these images are seen are described in the next article.

The following is a general construction for determining the image of an object formed by a spherical mirror. Let A B (Figs. 46, 48, 49) represent an object placed in front of the mirror P M. Consider the ray A M coming from A and incident on the mirror normally at M. Its direction is obtained by joining A C, and, if necessary, producing the line to cut the mirror in M. The reflected ray M A travels back along the path of the incident ray (Art. **37**), and the image of A lies somewhere on this path. Again the ray A P, drawn parallel to the principal axis, is reflected along P F (Art. **37**), and the image of A lies on this line also. Hence the image of A is found at A′, the intersection of the lines M A and P F. A third ray A P′ through F may also be drawn. It is reflected along P′ A′ parallel to the principal axis. Similarly, an image of B is formed at B′, and images of points lying between A and B are formed at corresponding points between A′ and B′, and therefore A′ B′ is the complete image of A B.

In connection with the formation of images the following four points have to be considered:—

1. *Relative position of image and object.* This has been fully considered in preceding article; the reasoning there employed is applicable whether the luminous point P be an isolated point, or a point on an object of finite size.

2. *Whether the image is real or virtual.* Whenever the image appears behind the mirror, it must necessarily be virtual; hence, it is only necessary to know the position of an image to decide whether it is real or virtual.

3. *Whether the image is inverted or erect.* It is evident, from Fig. 46, that when object and image are on opposite sides of C, the latter is inverted because of the crossing of the rays passing through C. Hence, if the relative positions of object and image are known, this point is easily decided. It may be remarked that the image of a real object is always inverted if real, and erect if virtual.

4. *Relative size of image and object.* The ratio of the

linear dimensions of the image to the corresponding linear dimensions of the object is called the **magnification**. In Fig. 46 the magnification is equal to the ratio of A′ B′ to A B. When the image is erect the magnification is taken as positive, and when inverted it is regarded as negative. The magnification is often written as $\frac{\text{image}}{\text{object}}$ or **m**, and we shall now proceed to express **m** in terms u v and f, using Fig. 46 for this purpose, as in that diagram all these quantities are positive.

(*a*) In the triangles A′ B′ C and A B C we have

$$\frac{A'B'}{AB} = \frac{CX'}{CX} = \frac{c'}{c}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from C}}{\text{Distance of object from C}} = \frac{c}{c} = -\frac{r-v}{u-r}, \text{....(2 a)}$$

the negative sign being placed in accordance with the above reasons. The negative sign is not needed before $\frac{c'}{c}$, for if the image is inverted c' is of opposite sign to c.

(*b*) A ray A N incident at the pole of the mirror is reflected to A′ since A′ is the image of A. The angles A N X and A′ N X′ are thus equal, and the right-angled triangles A N X, A′ N X′ are similar.

Hence—

$$\frac{A'X'}{AX} = \frac{NX'}{NX}.$$

Similarly—

$$\frac{B'X'}{BX} = \frac{NX'}{NX};$$

therefore by addition—

$$\frac{A'B'}{AB} = \frac{NX'}{NX} = \frac{v}{u}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}} = -\frac{v}{u}, \text{ .... (2b)}$$

the negative sign being inserted, since the image is inverted.

(*c*) The triangle A F X is similar to the approximate triangle P′ F N.

$$\therefore \frac{\mathrm{P'N}}{\mathrm{AX}} = \frac{\mathrm{NF}}{\mathrm{XF}} = \frac{f}{u-f}.$$

But—

$$\frac{\mathrm{P'N}}{\mathrm{AX}} = \frac{\mathrm{A'X'}}{\mathrm{AX}} = \frac{\mathrm{A'B'}}{\mathrm{AB}} \text{ (by the above)};$$

$$\therefore \frac{\mathrm{A'B'}}{\mathrm{AB}} = \frac{f}{u-f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2c)}$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Focal length of mirror}}{\text{Distance of object from focus}} = -\frac{f}{u-f},$$

the negative sign being again introduced.

(*d*) The triangle A′ F X′ is similar to the approximate triangle P N F.

$$\therefore \frac{\mathrm{A'X'}}{\mathrm{PN}} = \frac{\mathrm{FX'}}{\mathrm{NF}}.$$

But—

$$\mathrm{PN} = \mathrm{AX}$$

$$\therefore \frac{\mathrm{A'X'}}{\mathrm{AX}} = \frac{\mathrm{FX'}}{\mathrm{NF}} = \frac{v-f}{f}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from focus}}{\text{Focal length of mirror}} = -\frac{v-f}{f} \quad \ldots\ldots\ldots \text{(2d)}$$

since the image is inverted.

In this way we get four different but not independent relations between the linear dimensions of the image and object. These are—

$$\mathrm{m} = \frac{\text{Image}}{\text{Object}} = \frac{c'}{c} = -\frac{v}{u} = -\frac{f}{u-f} = -\frac{v-f}{f}. \qquad \text{(3)}.$$

That these four expressions are equal can also be very simply proved from the general equation—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

For example, to prove the equality of second and third, multiply the general equation by u.

$$\frac{u}{v} + 1 = \frac{u}{f}$$

$$\therefore \frac{u}{v} = \frac{u}{f} - 1 = \frac{u-f}{f}$$

$$i.e.\ \frac{v}{u} = \frac{f}{u-f}.$$

The proportions expressed by the above equations apply to linear dimensions only; for relative areas we have—

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{c'}{c}\right)^2 = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{u-f}\right)^2 = \left(\frac{v-f}{f}\right)^2.$$

It thus appears that if we know the positions of the object and of its image we can completely determine the nature of the image. The results of Art. **40**, as there summarised, are therefore of great importance, and for this reason we give below, with figures, the cases for a luminous object of finite size, corresponding to I., 1, 2, 3, 4, and II., 1 of that article.

I. Concave mirror.

(1) Object between C and infinity in front of mirror. The image lies between C and F, and is *real*, *inverted*, and *diminished*. (Fig. 46.)

(2) Object C B at C. The image, C B′, at C, is *real*, *inverted*, and of *same size* as object. (Fig. 47.)

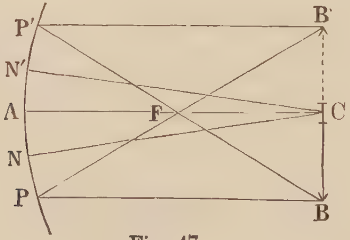

Fig. 47.

The construction for this image should be noticed. Any two rays C N, C N′ coming from C are normal to the mirror, and therefore, on reflection, again intersect at C. That is, the image of C is formed at C. B′ the image of B is found in the usual way.

(3) Object between C and F. Image between C and infinity in front of the mirror. If A′ B′ of Fig. 46 be supposed to represent the object, then A B represents its image, and the figure illustrates the case we are now considering. The image is *real*, *inverted*, and *magnified*.

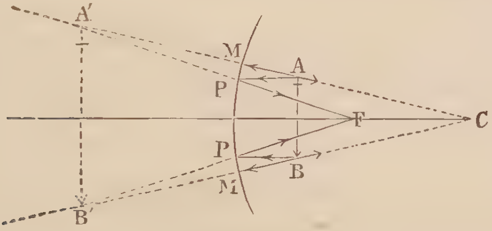

Fig. 48.

(4) Object between F and the pole. The image is *behind* the mirror, between infinity and the pole, and is *virtual*, *erect*, and *magnified*. (Fig. 48.)

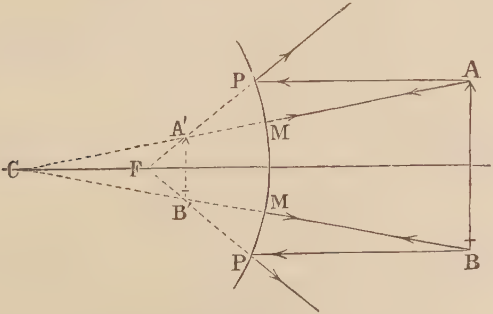

Fig. 49.

In the limit, when the object is at the pole, the image is also at the pole, and coincides with the object in position and size (Art. 40, I., 5).

II. Convex mirror.

Object in front of mirror between infinity and the pole. The image lies between F and the pole, and is *virtual, erect,* and *diminished.* (Fig. 49.)

Examples. Formulae 1 and 2 (*b*) are the most important formulae of this chapter.

Formulae 2 (*a*) and 2 (*b*) should be learnt *in words* (Art. 41) ; 2 (*c*) and 2 (*d*) are not important, but are sometimes very convenient. The different forms of formula 2 may be remembered by noticing that "image" and "v" are associated, as are also "object" and "u."

Sign need not be considered in connection with formulae 2 if the ratios be learnt *in words.* But if learnt as formulae involving u, v, f, c, and c', then the signs must be considered, just as in any other case, and the interpretations of the results given in Art. 41 must be remembered.

When the magnification is one of the data of a problem, great attention must be paid to this point. See Example 3 *below.*

If on substitution $\frac{\text{image}}{\text{object}}$ is found to be positive the image is erect, and, if negative, inverted.

A convex mirror gives only virtual images, and these are always erect. A concave mirror gives inverted real images and erect virtual images.

1. In the case of Example 1, page 68, find ratio of the size of the image to that of the object.

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{-30}{15} = +2$$

or

$$\frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{30}{15-30} = +2.$$

2. In the case of Example 3, page 68, find size of the image.

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = +\frac{7{\cdot}5}{20}.$$

That is, image is *virtual ;* and—

$$\frac{\text{Length of image}}{3 \text{ cm.}} = \frac{3}{8}.$$

$$\therefore \text{Length of image} = \frac{9}{8} = 1\tfrac{1}{8} \text{ cms.}$$

We might have used the formula

$$m = -\frac{v-f}{f}.$$

This gives us
$$m = -\frac{-7\cdot5+12}{-12}$$
$$= \frac{4\cdot5}{12} = \frac{3}{8},$$

the same result as before.

3. A small object on the axis of a concave mirror, at a distance of 10 inches from it, gives an image which is four times its own length. Find the focal length of the mirror.

Here, applying the third relation of Art. 41, we have—

$$m = \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f}.$$

If image is *real* it is inverted and $\therefore$ m is equal to -4

$$\therefore\ -4 = -\frac{f}{10-f}$$
$$\therefore\ 40 - 4f = f \text{ or } f = 8 \text{ ins.}$$

If image is *virtual* it is erect and therefore m is equal to $+4$.

$$\therefore\ 4 = -\frac{f}{10-f}$$
$$\therefore\ 40 - 4f = -f \text{ or } f = 13\tfrac{1}{3} \text{ ins.}$$

4. A gas flame is placed at a distance of 8 feet from the wall of a room. Find the radius of curvature of a concave spherical mirror, and where it must be placed in order that it may produce, on the wall, an image of the gas flame magnified threefold.

Here, if x denote the distance of the mirror from the gas flame, we have—

$$u = x;\ v = x + 8$$

and—

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{x+8}{x} = -3 \ (\textit{i.e.}\ \text{Image is inverted})$$
$$\therefore\ 3x = x + 8$$
$$\therefore\ x = 4$$

And—

$$\frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f}, \quad \therefore -3 = -\frac{f}{4-f}, \text{ or}$$
$$12 - 3f = f.$$
$$\therefore\ f = 3 \text{ and } r = 6.$$

Or, after determining $x = 4$, we may employ the formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ instead of 2 (c)}$$

$$\frac{2}{r} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}.$$

$$\therefore\ r = 6.$$

Hence the mirror must be placed 4 feet from the gas flame—that is, 12 feet from the wall—and its radius of curvature should be 6 feet.

5. A square piece of cardboard of 1 inch side is placed at right angles to the principal axis of a concave mirror of 18 inches focal length. At what distance from the mirror must it be placed in order that an image, 9 square inches in area, may be formed?

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{f}{u - f}\right)^2.$$

$$\therefore\ \frac{9}{1} = \left(\frac{18}{u - 18}\right)^2.$$

$$\therefore\ 3 = \pm\,\frac{18}{u - 18}.$$

$$\therefore\ u = 24 \text{ or } 12.$$

That is, the object may be placed 24 inches in front of the mirror, or 12 inches in front of the mirror. In the former case the image is *real* and *inverted*; in the latter it is *virtual* and *erect*.

42. Exercise.—**To draw the rays by which an eye sees an image of an object formed by reflection at a spherical mirror.** If the object is near the principal axis, the image will be near the axis, and, therefore, also the eye must not be far removed from the axis. Let M M (Fig. 50) be a concave mirror, A B an object placed in front of it, A′ B′ the real image of A B, and E the position of the eye. Now this image is only the locus of intersection of reflected rays,

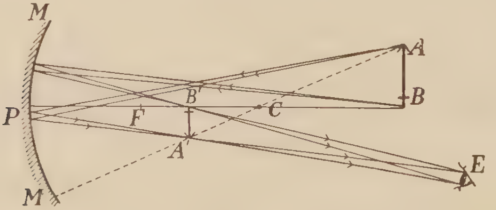

Fig. 50.

and hence is not self-luminous, so that it can be seen only by those rays which originally come from the object, and, passing through the image, enter the eye.

Thus, to depict the rays by which E sees A′, draw a pencil of rays diverging from A′ and entering E. Produce the rays backwards to meet the mirror at P, and join the points of intersection to A. The rays by which A is seen are included in the incident pencil A P and the reflected pencil P A′ E. The same construction can be applied to other points of the image and object, and a similar construction holds for the visual rays by which the virtual image of an object placed in front of a concave or convex mirror (Fig. 51) is seen.

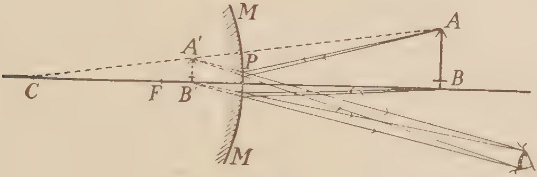

Fig. 51.

Repeat Exps. **15** and **16** in the light of Figs. 50 and 51.

43. Spherical aberration. In dealing with the laws of reflection from concave spherical mirrors, we have, up to now, limited the discussion to mirrors of small aperture, and we have learnt that parallel rays falling on such a mirror are all brought together at one point—the principal focus; and that rays diverging from any luminous point are all brought together at one point—the conjugate focus. In order to explain the necessity of this limitation we will now consider the case of a concave mirror of a very large aperture, that is, forming a very large segment of a sphere. It will well repay the student to do for himself, on a larger scale and drawing many more rays, what has been done in Fig. 52. Draw a large segment of a circle. Through its centre draw the diameter C A, and parallel with this a number of equidistant straight lines to

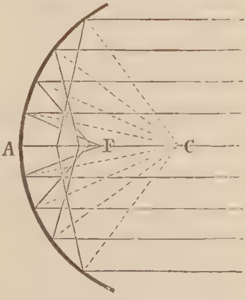

Fig. 52.

represent rays of light in a parallel beam falling on a concave mirror. From C draw dotted radii to every point at which a ray is incident on the mirror. These are the normals to those points. Then from those points, and on the inner sides of the normals, set off angles exactly equal to those on the other sides, and draw straight lines at these angles to represent the reflected rays. It will then be seen that the rays nearest the axial ray cut that ray after reflection at a point as near as possible halfway between C and A. A pair of rays a little further from the axis will be found to intersect the axis a trifle behind this point. The next pair of rays intersect the axis after reflection considerably behind F, and the next pair still further behind. Any such wandering of the marginal rays from the focus of the central rays is called *aberration*, and this particular case being due to the form (spherical) of curve employed, is called *spherical aberration.*

44. Caustic. Fig. 52 shows, and the student's diagram with three or four times as many rays does so more clearly, that all the reflected rays lie within an area bounded at the back by the mirror, and in front by a double curve called the *caustic curve.* This curve is very bright, especially at its vertex, which coincides with the principal focus. Such a curve may be clearly seen on the surface of milk, when a tumbler is about three-quarters filled; a candle being placed so that the inner rim of the glass reflects its light down upon the milk.

45. Diaphragms or Stops. For all optical purposes spherical aberration is an important defect, and the way it is kept within allowable limits is by using mirrors of very small aperture, or by screening the margin of the mirror by an opaque plate called a *diaphragm*, with a central hole in it. Of course the more the mirror is *stopped down* the sharper becomes the definition, but the loss of light becomes at last very serious. It must be noted, too, that these means do not correct, but simply lessen, the defect, which indeed is inseparable from the spherical form.

46. The optical bench. This apparatus is of such frequent use in optical measurements that it is advisable, at this stage, to consider briefly its construction and method of use. The details of construction are very varied for benches of different make, but the principle is the same for all, and is sufficiently illustrated by the simple apparatus of Fig. 53. It consists of a thick base-board, B B, about 5 feet long, along the top of which is inlaid a scale reading to centimetres and millimetres. Little stools or trestles are made to slide with gentle friction along this. To two of these, S_1, S_2, are fixed white mill-board screens, A and C, and to another, S_3, two adjustable arms grooved on the inside. The edges of the mirror are placed within the grooves, and an elastic band is placed around them to keep them in position. Another, S_4, supports a measuring rod, P, of known length provided with pointed ends. A part of the base of S_4 is removed as shown, and then furnished with a mark which slides over the graduations on the scale. The position of S_4 is thus easily noted. When measuring distances between different surfaces the ends of P are brought into contact with these surfaces, and the readings are taken from the mark on the base. If the same end of the rod touches the two surfaces the distance between the surfaces is equal to the difference in readings of S_4. If, however, opposite ends of P touch the surfaces in turn the distance between them is equal to the difference in readings of S_4, plus or minus the length of the measuring rod according as P is between or beyond the surfaces. A circular hole 1 cm. in diameter is cut in the middle of A. It is then either provided with cross-wires by boring little holes around it and threading fine wire through them, or with a piece of fine gauze, which may be attached to A by gummed paper. The cross-wires are strongly illuminated at the back by an oil flame, a gas flame, an incandescent mantle, or an electric lamp. The cross-wires or gauze act as object, and being in a vertical plane measurement of *u* can be accurately made. It is convenient to be able to slide C along the groove in S_2. If the screen is then in the way of the light it may be slid to one side. (See Fig. 53.)

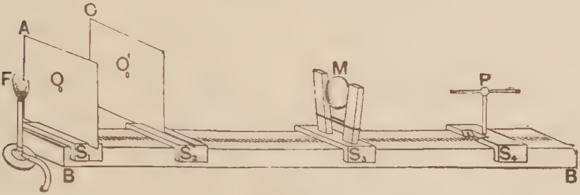

Fig. 53.

47. Experimental determination of the radius of curvature, and the focal length of a mirror.

I. Concave mirrors.—1. The simplest method of determining the focal length is to allow a beam of parallel light to be incident on the mirror in a direction parallel to the principal axis, and then to measure the distance of the focus of the reflected beam from the mirror.

Exp. 18. For this purpose mount the mirror in a suitable stand or clip, with its axis parallel to a graduated bar of wood, along which the stand slides. At the zero end of the bar, and at right angles to its length, fix a paper or millboard screen with its centre approximately on the same level as the principal axis of the mirror. Point this arrangement toward the sun, or some other well-illuminated distant object, and adjust the position of the mirror by the method of oscillations (Exp. 6) until a clearly defined image of the object chosen is formed on the screen. The distance between the mirror and screen as indicated by the graduations on the bar is the focal length, for if the object is sufficiently distant the image is practically at the principal focus of the mirror.

2. Art. **41** proves that when an object is placed at the centre of curvature of a concave mirror, the image is also at the centre, but in an inverted position.

Exp. 19. Fix a short polished needle vertically, and point upwards in a wooden stand or clip, and place it in front of the mirror, so that the point of the needle is on the principal axis of the mirror.

Unless the needle be placed too close to the mirror, an inverted image may be seen by an eye placed near the principal axis, at some distance from the mirror (as in Fig. 50). By repeated trials adjust the position of the needle until its point coincides with the point of the inverted image for *several positions of the eyes.*

The point of the needle is now at the centre of the mirror, and hence the radius of curvature is obtained by measuring the distance between the pole of the mirror and the point of the needle. The focal length is equal to half this distance. If the distance is small, measure it by means of a pair of compasses and a scale; but, if large, use a wooden rod, pointed at both ends and of adjustable length, or fix the mirror and needle in two of the uprights of the optical bench, and make the measurements as described in Art. **46**.

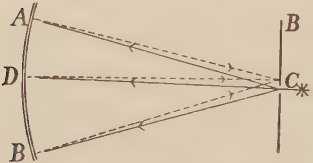

Fig. 54.

Exp. 20. Using an optical bench and the form of object described in Art. 46, adjust the mirror until an image of the gauze is focussed on the screen alongside the gauze itself (Fig. 54). Since the rays thus return very nearly along their former paths, C is practically the centre of the surface A D B. Measure C D as in Exp. 19.

3. The general formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, may also be employed.

Exp. 21. Mount the mirror in an optical bench in a slightly slanting position, and receive the image P on a screen placed as in Fig. 55, so as not to interfere too much with the rays from O. Measure u and v, and calculate f. Repeat for different distances.

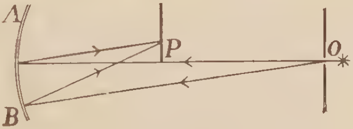

Fig. 55.

II. Convex mirrors.—

1. The radius may be determined by means of a parallax method similar to that of Exp. 19.

Exp. 22. Mount the mirror on the optical bench as usual, and place before it a well-defined object, such as a white thermometer tube. An image can be seen; of course it is virtual. Place a pin on the stand behind the mirror, so that it is just visible over the top, and adjust its distance from the mirror till it stands as nearly as possible over the image of the thermometer tube for all positions of the eye. Then the distance of tube from mirror = u, and the distance of pin from mirror = v, and f can be found from the general formula.

A similar method can be used for a concave mirror, the tube being placed so near it as to have a virtual image. But unless the mirror has a very large radius of curvature this method is not to be recommended.

2. All other optical methods require the use of lenses, hence a description of the methods employed is postponed to Art. 87.

EXERCISES IV.

1. Given a concave mirror whose focal length is 12 inches, where would you place a candle flame in order that the image of it, formed by the mirror, may be (1) real, (2) virtual?

2. A concave spherical mirror is so placed that a candle flame is situated on its principal axis at a distance of 18 inches from its surface. An inverted image, three times as long as the candle flame itself, is seen sharply defined on the wall. What is the focal length of the mirror?

3. Prove that if an object is placed at a distance of $3f$ in front of a concave mirror (of focal length f), then the image is one-half the size of the object.

4. A small object on the axis of a concave mirror, at a distance of 16 inches from it, produces a *real* image which is three times its own size. Find the focal length of the mirror.

5. A small object 0·1 inch long is placed at a distance of 3 feet from a convex mirror of 12 inches focal length. What is the length of the image and its distance from the mirror?

6. A penny is held 8 inches in front of a convex mirror of 1 foot radius. Where will its image be, and what will be its diameter compared with that of the penny?

7. An object is held in front of a convex mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

8. An image produced by a convex mirror of focal length f is $1/r$th the size of the object. Show that the distance of the object from the mirror is $(r-1)f$.

9. A penny is held at a distance of 12 inches from a concave mirror whose focal length is 4 inches, and then at the same distance from a convex mirror of the same focal length. Compare the sizes of the images.

10. A concave mirror whose focal length is 1 foot is placed at a distance of 4 feet from a screen. Where must a candle be placed so that its image may be distinctly formed on the screen?

11. A room is 25 feet wide, and a gas jet is placed at a distance of 1 foot from one of the walls. What must be the focal length of a spherical mirror attached to the wall behind the gas jet so that its image may be formed on the opposite wall?

12. In the previous question where ought the mirror to have been placed if its focal length had been half a foot?

13. A candle flame is placed at a distance of 3 feet from a concave mirror formed of a portion of a sphere, the diameter of which is 3 feet. Determine the nature and position of the image of the candle flame produced by the mirror, and state whether it is erect or inverted.

14. An object 6 inches long is placed symmetrically on the axis of a convex spherical mirror, and at a distance of 12 inches from it. The image formed is found to be 2 inches long. What is the focal length of the mirror?

15. A small object is placed in front of a concave spherical mirror of 6 inches radius at a distance of 4 inches from the surface of the mirror. Where will its image be situated? Will it be erect or inverted? and what will its dimensions be compared with those of the object? Where must the object be that the image may be of the same size?

16. The radius of a convex mirror is 6 inches. If the linear dimensions of the object are twice those of the image, where must each be situated?

EXAMINATION QUESTIONS.

1. State the laws of the reflection of light. Do they hold for a curved reflecting surface? How may they be verified experimentally?

2. Explain the formation of images in a plane mirror, and prove that the distance of the image is equal to that of the object from the mirror.

3. A plane mirror revolves about an axis. Explain a method of ascertaining experimentally whether or not the axis is perpendicular to the surface of the mirror.

4. Two plane mirrors are inclined at an angle of 60°; trace the path of a pencil of rays proceeding from a luminous point between the mirrors to the eye, after undergoing one reflection at the surface of each mirror.

5. Two plane mirrors are inclined at an angle of 30°; trace the path of a pencil of rays proceeding from a luminous point between the mirrors to the eye, after undergoing two reflections at the surface of one mirror and one reflection at the surface of the other.

6. When a ray of light falls upon a rotating mirror, show that the reflected ray turns twice as fast as the mirror.

7. The angle between two plane mirrors is one-fifth of four right angles, and a luminous point is placed midway between them. Make a diagram showing the images that are formed, and explain carefully how each one is formed.

Trace a ray by which an eye placed midway between the mirrors and twice as far as the luminous point from their intersection sees an image formed by two reflections.

8. A ray of light is reflected successively at two plane mirrors, the plane of incidence being perpendicular to the line of intersection of the mirrors: prove that when the mirrors are at right angles to each other the final direction of the ray is parallel to its original direction.

9. Assuming the laws of the ordinary reflection of light, find the position of the image of an object in front of a plane mirror. What are the limits of the position of the object (the mirror being supposed fixed), so that an image of it may be formed by the mirror?

10. What is meant by the *image* of a luminous point?

A luminous point is placed in front of a plane mirror. Determine from the laws of reflection of light the position of its image.

How would you determine the position of the image by experiment?

11. Two mirrors are placed parallel to one another, and a luminous point is placed midway between them. Show how to draw accurately the path of a ray of light which, after undergoing three reflections at one mirror and four at the other, enters an eye also placed midway between the mirrors, but at some distance from the source of light.

12. On a moonlight night, when the surface of the sea is covered with small ripples, instead of an *image* of the moon being seen in the sea, a long band of light is observed on the surface of the sea extending towards the point which is vertically beneath the moon. Account for this phenomenon in accordance with the laws of reflection, illustrating your explanation by a figure.

13. Each of the four walls of a rectangular room is a plane mirror. Trace the course of a ray of light from a candle in the room, but not at its centre, which, after reflection at each of the walls, returns to the candle. Is there more than one ray satisfying the above conditions?

14. Sketch a concave spherical mirror exhibiting a distant luminous object, and showing the position and nature of the image of this object given by the mirror.

15. State the laws of reflection of light.

A candle is placed in front of a concave spherical mirror of radius 6 feet and at a distance of 9 feet from it. Draw the course of at least 3 rays from a point on the candle, and find the position and size of the image.

16. An electric arc is placed in front of a concave spherical mirror whose radius is 1 metre and at a distance of 50 cms. from the mirror, Trace the course of any three rays.

17. Show how to find the position of the image of an arrow placed in front of a concave spherical mirror. Explain when the image is erect and when inverted.

18. Explain the formation of images by a concave cylindrical mirror. Find the relation between the distances of the two conjugate foci from the mirror. What is the position of the image of a point which is at the distance of the diameter from the reflecting surface of the cylinder?

19. If a small object on the principal axis of a concave mirror is gradually moved up to the mirror from a point at a considerable distance, show what will be the simultaneous changes in the position and size of the image.

20. A source of light is placed on the axis of a hemispherical mirror midway between the mirror and its centre of curvature. Draw a careful diagram to show in detail how the rays of light from the source proceed after reflection from the mirror.

21. Show, by very carefully drawn figures, that as an object moves from a great distance up to a convex mirror the image moves from a point half-way between the mirror and its centre up to the mirror, and show that the image is always erect and less than the object.

22. An arrow 2 inches long is held 4 inches from a convex mirror of 8 inches radius. Where is the image?

Draw a figure as nearly as you can to scale showing the relative positions and sizes of object and image, and the complete course of the rays by which an eye, a little above the axis, sees the image of the lower end of the arrow.

23. It is required to find the radius of curvature of a spherical mirror of such a curvature that, when placed at a distance p from a point P, it will approximately concentrate at that point the waves of light which originate at a point Q, situated at a distance q away from P and on the other side of it from the mirror.

24. Given a concave spherical mirror, how could you find its radius of curvature by optical means alone, and without resorting to geometrical operations?

CHAPTER VI.

REFRACTION AT PLANE SURFACES.

48. Refraction. We have seen in Art. 7 that a ray of light travels in a straight line so long as its course lies in the same homogeneous medium, but when it passes from one medium into another it undergoes a change of direction at the surface of separation of the two media. This change of direction is called **refraction**. A simple experiment illustrating this has already been described (Art. 7).

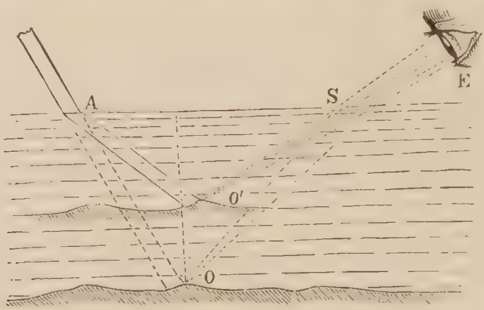

Fig. 56.

Exp. 23. Partly immerse a stick in water in an oblique position; it appears bent at the surface of the water (Fig. 56). This is due to the refraction of the rays coming from points of the stick below the surface of the water. For example, rays coming from O are refracted at S in passing from the water to the air, and appear to come from O′. Similarly, other points between O and A appear, at E, to lie between O′ and A, and thus the portion O A of the stick appears bent into the position O′ A.

For similar reasons a pool of water appears shallower than it really is, and small air bubbles in solid glass objects appear nearer the surface than they actually are (Art. **57**).

49. Angles of incidence and refraction. Let A O (Fig. 57) represent a ray of light incident at O on the surface of separation of the media M and M′, and let O B represent the refracted ray. Then, if N O N′ be the normal to the surface of O, the angle A O N is the *angle of incidence*, and B O N′ is the corresponding *angle of refraction*. The laws of refraction, as established by experiment, refer to the relative position and magnitude of these angles, and may be stated thus:—

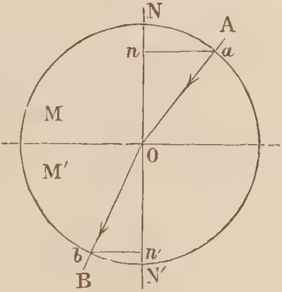

Fig. 57.

(i) *The angles of incidence and refraction lie in the same plane—that is, the incident and refracted rays, and the normal at the point of incidence, all lie in the same plane.*

(ii) *For the same two media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always the same.*

This law is generally known as **the law of sines**, and the above ratio is called the relative refractive index between the two media. Without employing the term *sine*, this law may be explained by a geometrical construction. With centre O (Fig. 57) and any radius O *a* describe the circle *a* N *b* N′, cutting O A and O B in *a* and *b*. From *a* and *b* drop perpendiculars *a n* and *b n′* on the normal N N′. Then the law may be expressed by stating that, for the same two media, the ratio $\frac{a\,n}{b\,n'}$, is constant.

Exp. 24. To verify the laws of refraction and to illustrate further effects. Fix a rectangular block of glass A B (Fig. 58) upon a sheet of cartridge paper, and mark in the outline with a sharp pencil. Fix a pin, P, into the paper, and close up to the glass. Arrange other pins, P_1 P_2 P_3 P_4, at convenient distances from one another and from P. With the eye on a level with the block and looking through it towards P P_1, place two pins Q_1, R_1, Q_1 being in contact with the glass and R_1 some distance out, so that R_1 Q_1 P P_1 appear in a straight line. Do the same with R_2 Q_2 P P_2, R_3 Q_3 P P_3, etc.

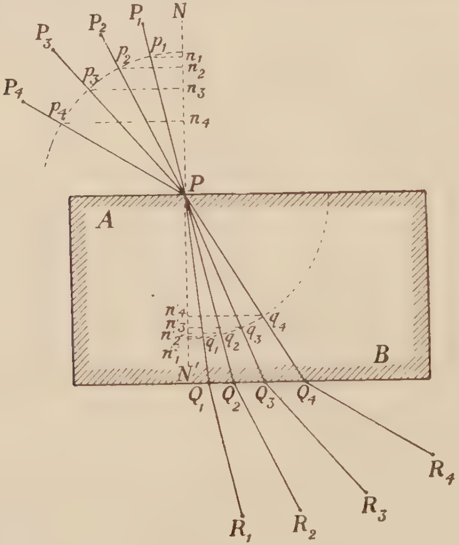

Fig. 58.

Remove A B and join up the pinpricks by straight lines. Draw the normal N N^1 at P.

The rays P_1P, P_2P, P_3P, P_4P, . . . are incident rays; the rays P Q_1, P Q_2, P Q_3, P Q_4, . . . are refracted rays; and the rays Q_1 R_1, Q_2 R_2, Q_3 R_3, Q_4 R_4, . . . are emergent rays. The angles P_1P N, P_2P N, P_3P N, . . . are angles of incidence on face P A, and the angles Q_1P N′, Q_2P N′, Q_3P N′, . . . are the corresponding angles of refraction. Measure these angles with a protractor. Look up the values of their sines in a book of tables.

Show that—

$$\frac{\sin P_1PN}{\sin Q_1PN'} = \frac{\sin P_2PN}{\sin Q_2PN'} = \frac{\sin P_3QN}{\sin Q_3PN'} = \ldots = \text{a constant, } \mu \text{ say.}$$

If a protractor is not available describe a circle with P as centre cutting the rays in $p_1p_2p_3 \ldots q_1q_2q_3 \ldots$ Draw perpendiculars p_1n_1, p_2n_2, $p_3n_3 \ldots q_1n_1'$, q_2n_2', $q_3n_3' \ldots$ to N N′. Measure these with a pair of dividers and a diagonal scale and show that

$$\frac{p_1n_1}{q_1n_1'} = \frac{p_2n_2}{q_2n_2'} = \frac{p_3n_3}{q_3n_3'} = \ldots = \text{the same constant, } \mu.$$

For glass μ will be about 1·5.

Observe that when light enters glass from air the ray is bent towards the normal, and that when it leaves glass for air it is bent away from the normal. The same is true if the glass is replaced by any other transparent body of greater density than air.

The deviation of the ray P_1P on entering the glass is the difference between the angles P_1PN, Q_1PN'. Calculate this, and show that for the other rays the deviation **increases with the angle of incidence.** Note that when the angle of incidence is zero the angle of refraction is also zero and no deviation is produced.

Also observe that P_1P, P_2P, P_3P, . . . are parallel to Q_1R_1, Q_2R_2, Q_3R_3, . . ., showing that the directions of the rays have not been changed by passing through the parallel-sided block, but only that the rays have been **shifted laterally** by a distance which increases with the angle of incidence.

This lateral shifting explains the **apparent displacement of a body seen through a plate with parallel sides.** It is evident from the experiment that the amount of lateral displacement increases with (1) the thickness of the plate and (2) the obliquity of the incident ray. It is also increased if a block of substance of greater refractive index is employed. This is well shown by the following experiment:—

Exp. 25. Take a flat glass cell about 4 ins. high, 4 ins. long, 1 in. broad. Fill it one-third full with bisulphide of carbon, and one-third full of water. Look obliquely through it at a stick or retort rod as in the last experiment, and observe the varying displacement of the rod in the three parts of the cell.

50. Refractive indices. We have seen that, when a ray of light is refracted from one medium, *a*, into another, *b*, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant. This ratio is the

relative index of refraction from the medium a into the medium b. That is, if ${}_a\mu_b$ represent this index, and if ϕ and ϕ' denote respectively the angles of incidence and refraction, we may write—

$$ {}_a\mu_b = \frac{\sin\phi}{\sin\phi'}. \qquad (a.)$$

It has been established by experiment that the path of a ray of light is reversible—that is, if, in Fig. 57, B O be taken to represent the incident ray, then O A will be the path of the refracted ray. This fact is evidently expressed by writing—

$$ {}_b\mu_a = \frac{\sin\phi'}{\sin\phi}. \qquad (b.)$$

From (a) and (b) we have—

$$ {}_a\mu_b \cdot {}_b\mu_a = \frac{\sin\phi}{\sin\phi'} \cdot \frac{\sin\phi'}{\sin\phi} = 1.$$

That is—

$$ {}_a\mu_b = \frac{1}{{}_b\mu_a}, \text{ or } {}_b\mu_a = \frac{1}{{}_a\mu_b}. \qquad (1.)$$

This result may be stated in words by saying, that if ${}_a\mu_b$ denotes the index of refraction from a to b, then $\frac{1}{{}_a\mu_b}$ denotes the index of refraction from b to a. For example, if the index of refraction from air to water be $\frac{4}{3}$, then the index of refraction from water to air is $\frac{3}{4}$.

When a ray of light is refracted from vacuum into any other medium, the index of refraction from vacuum into that medium is called the **absolute refractive index** or the **refractive index** of the medium.

If a ray of light pass from a given medium, through a layer of another medium bounded by parallel planes, into the medium in which it was originally travelling, it is known, from experiment, that the initial and final directions of the ray are parallel. This may either be taken as an experimental fact, or deduced from results already obtained from experimental data (*see* Exp. **24**).

Thus, let A A and B B (Fig. 59) represent the parallel surfaces of separation of a layer of the medium b from a, and let R N, R′ N′ represent the path of a ray travelling from *a* through *b* into *a* again. Then, it is evident that the angles R′ N′ *n′* and R N *n* are equal, for they have respectively the same relation to the equal angles O′ N′ N and O N N′. Hence R′ N′ is parallel to R N, but is not in the same straight line with it. It follows from this that when a ray of light passes from one medium through any number of layers of different media, having parallel surfaces of separation, back into the same medium, then the initial and final directions of the ray are parallel.

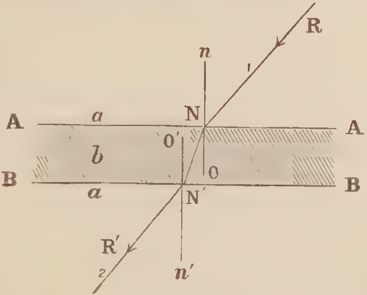

Fig. 59.

Consider the case for the three media *a*, *b*, *c*, shown in Fig. 60. Here—

$$ {}_a\mu_b = \frac{\sin \phi_1}{\sin \phi_2} $$

$$ {}_b\mu_c = \frac{\sin \phi_2}{\sin \phi_3} $$

$$ {}_c\mu_a = \frac{\sin \phi_3}{\sin \phi_1} $$

$$ \therefore\ {}_a\mu_b \cdot {}_b\mu_c \cdot {}_c\mu_a = 1. $$

$$ \therefore\ {}_a\mu_b \cdot {}_b\mu_c = \frac{1}{{}_c\mu_a} = {}_a\mu_c \text{ [by 1 above].} $$

That is—

$$ {}_a\mu_c = {}_a\mu_b \cdot {}_b\mu_c. \qquad (2.)^* $$

This is an important relation, and enables us to determine the relative index of refraction from *a* to *c*, given the indices of refraction from *a* to *b* and from *b* to *c*. For example, if the index of refraction from air to glass, ${}_a\mu_g$, is $\frac{3}{2}$, and that from air to water, ${}_a\mu_w$, is $\frac{4}{3}$, then the index of refraction from water to glass, ${}_w\mu_g$, is given by—

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* This formula is easily remembered by noticing that *a* and *c* are the initial and final suffixes on *each* side. Compare the suffixes *w*, *g*, on next page.

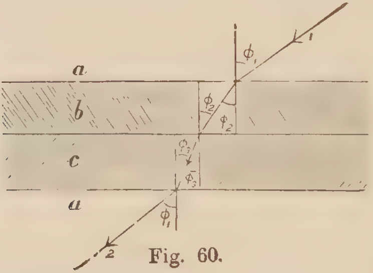

Fig. 60.

$$ {}_w\mu_g = {}_w\mu_a \cdot {}_a\mu_g = \frac{1}{{}_a\mu_w} \times {}_a\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} . $$

That is—

$$ {}_w\mu_g = \tfrac{3}{2} \times \tfrac{3}{4} = \tfrac{9}{8}. $$

This formula also enables us to establish a relation between the relative refractive index for any two media and the absolute refractive indices of those media. Thus, if ${}_v\mu_a$ denote the absolute refractive index of the medium a, and ${}_v\mu_b$ that of b, then—

$$ {}_a\mu_b = {}_a\mu_v \cdot {}_v\mu_b $$

or—

$$ {}_a\mu_b = \frac{{}_v\mu_b}{{}_v\mu_a} . \qquad (8.) $$

That is, *the relative index of refraction from a to b is the ratio of the absolute refractive index of b to the absolute refractive index of a.**

It should here be noticed that, as a general rule, a ray of light in passing from one medium into a denser one is bent towards the normal, while in passing into a rarer medium it is bent away from the normal. This is equivalent to stating, that if the medium *b* is denser than *a*, then—

---

* Since the absolute refractive index of air is 1·0003 it will be seen that for practical purposes there is no appreciable difference between the values of the absolute refractive index of any other substance and of its refractive index relative to air.

$$ {}_a\mu_b > 1, \text{ and} $$

$$ \therefore\ {}_b\mu_a = \frac{1}{{}_a\mu_b} < 1, $$

which expresses the case for refraction from *b* into the rarer medium *a*. Since all media are denser than vacuum, it follows that all absolute refractive indices are greater than unity.*

So far we have considered the index of refraction merely as a geometrical relation, established by experiment, between the directions of the incident and refracted rays. When considered in connection with the undulatory theory of light, a definite physical meaning can, however, be attached to this constant. It can be shown that the index of refraction from any medium *a* into another medium *b* is the ratio of the velocity of light in *a* to its velocity in *b*.

That is—

$$ {}_a\mu_b = \frac{V_a}{V_b}, $$

where V_a denotes the velocity of light in *a*, and V_b denotes the velocity of light in *b*.

This ratio differs for waves of different wave-length, being *greater* the *shorter* the wave-length; and, as difference of wave-length, in waves of light, corresponds to difference of *colour*, it follows that the value of the refractive index depends on the colour of the light which suffers refraction. The light having the *greatest wave-length* and *lowest refractive index* is of a deep *red* colour, and that of the *shortest wave-length* and *highest refractive index* is coloured *violet*. Between these two extremes the refractive index of a body increases as the wave-length of the light decreases, *i.e.* as the colour of the light shades off from red through orange, yellow, green, and blue to violet.

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* This law holds for familiar transparent media, but not for all media without exception. See the Table.

Example. The velocity of light in air is 300,000,000 metres per second. The refractive index of glass being $\frac{3}{2}$, find the velocity of light in glass.

Using the relation ${}_a\mu_b = \frac{V_a}{V_b}$,

$$V_b = \frac{300,000,000}{3/2}$$

$$= 200,000,000 \text{ metres per second.}$$

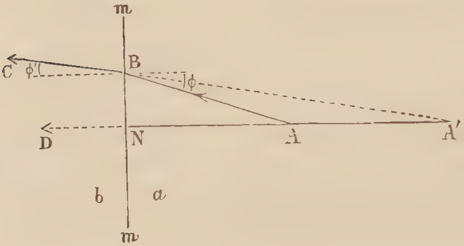

Fig. 61 (See Art. 51).

Table of Refractive Indices.*

(Mean Values.)

Diamond	2·60	Oil of turpentine	1·47
Iceland Spar	1·65	Sulphuric acid	1·43
Flint glass (heavy)	1·62	Alcohol	1·37
Rock-crystal	1·55	Ether	1·36
Crown glass (heavy)	1·53	Water (at 20° C.)	1·33
Plate glass	1·52	Carbonic acid gas	1·00045
Ice	1·31	Air	1·00029
Carbon disulphide (at 20°C.)	1·63	Hydrogen	1·00014

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* The student will find it convenient to remember the following approximate values :—

Refractive index for air and glass = $\frac{3}{2}$.

" " " water = $\frac{4}{3}$.

No others need be learnt. They will be given in the question if required.

51. Simple construction for incident and refracted rays. Let A B (Fig. 61) be any incident ray on a surface *m m* separating two media *a* and *b*. Draw any normal, N A A′, cutting the incident ray in A, and choose A′ on it so that $BA' = {}_a\mu_b BA$. Then join A′ B and produce it to C. B C is the refracted ray. For evidently

$$\frac{\sin\phi}{\sin\phi'} = \frac{\sin BAN}{\sin BA'N} = \frac{BN/BA}{BN/BA'} = \frac{BA'}{BA} = \mu, \text{ by construction.}$$

Hence the law is satisfied.

52. Critical angle. All possible values of an angle of incidence or refraction must evidently lie between 0° and 90°. Now, when a ray of light passes from a rarer into a denser medium, it is bent towards the normal—that is, the angle of refraction, ϕ', is less than the angle of incidence, ϕ, and therefore, whatever be the value of ϕ, between 0° and 90°, that of ϕ' must also lie between 0° and 90°, and consequently refraction is always possible. But, if a ray of light pass from a denser into a rarer medium, it is bent away from the normal, and the angle of refraction, ϕ', is greater than the angle of incidence, ϕ; so that, when ϕ passes a certain limit at which ϕ' becomes equal to 90°, refraction is no longer possible, and the incident ray is **totally reflected** at the surface of the rarer medium.

Let *m m* (Fig. 62) represent the surface of separation of any two media *a* and *b*, of which *b* is the rarer, and let I O represent a ray incident, at O, at a small angle I O N, and refracted along O R. As the angle of incidence increases and the incident ray takes the positions *a*, *b*, *c*, the angle of refraction also increases, and the refracted ray takes successively the corresponding positions *a′*, *b′*, *c′*. The angle of refraction being, however, greater than the angle of incidence, a position is reached at *c* where the angle of refraction R′ O N′ becomes equal to 90°, and the refracted ray, O R′, travels along the surface of separation of the media. The angle of incidence, I′ O N, at which this takes place is the **critical angle** for the media *a* and *b*. As the angle of incidence becomes greater than I′ O N the ray is no longer refracted into *b*, but is **totally reflected** from the surface

$m\,m$ in accordance with the ordinary laws of reflection. Hence, as the incident ray passes through the positions d, e, it is reflected from $m\,m$ along the corresponding paths, d', e'.

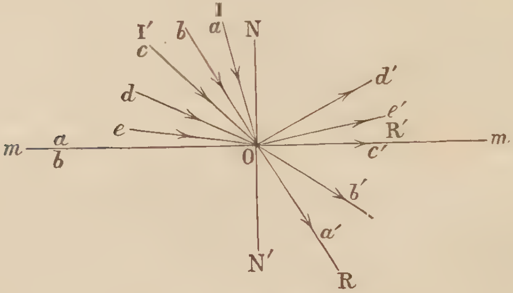

Fig. 62.

Hence, *when refraction takes place from a denser into a rarer medium, the angle of incidence, which corresponds to an angle of refraction of* 90°, *is called the critical angle for the given media.* At this angle refraction ceases and total reflection from the surface of separation of the media begins.

It should be noticed that for angles of incidence between 0° and the critical angle, only *a portion* of the light incident on the surface of the rarer medium is reflected at that surface, the remainder being refracted and scattered (Art. 20); but, for angles of incidence greater than the critical angle, the incident light is almost totally reflected, no portion of it being refracted.

The value of the critical angle is readily determined for any media when the relative index of refraction for those media is given. Thus, let ${}_a\mu_b$ denote the index of refraction from a to b, then, in notation used above, if θ denote the critical angle, we have—

$$ {}_a\mu_b = \frac{\sin\phi}{\sin\phi'}, = \frac{\sin\theta}{\sin 90^\circ} = \frac{\sin\theta}{1} = \sin\theta. $$

That is, the critical angle for refraction from a medium a

into a rarer medium b is the angle whose sine is the relative index of refraction from a to b, or—

$$ {}_a\theta_b = \sin^{-1}{}_a\mu_b. \qquad (4.)$$

This value of θ for air and water is about 48° 30′, and for air and glass it ranges from 38° to 41° according to the nature of the glass.

53. Total reflection. As we have seen in the preceding article, total reflection takes place when a ray of light, travelling in the denser of two media, is incident on the surface of separation at an angle greater than the critical angle of the media.

Simple illustrations of total reflection are often met with. For example, the edge of a crack in a pane of glass, seen obliquely, appears, by total reflection, as a brilliant reflecting surface.

Exp. 26. Place a coin in a round, clear, uncut wine-glass. Fill the glass a quarter full with water. Hold it above the head and look up at the water surface. Notice the reflection of the coin.

Exp. 27. Plunge an empty test-tube into a beaker of water; notice the metallic appearance of the part immersed. Fill the tube with water, and observe the disappearance of the lustre.

Exp. 28. Place a spoon in a tumbler of water. Hold it above the head and look up at the water surface. The part of the spoon out of the water cannot be seen at all; the part immersed is brilliantly reflected, as if by a silvered mirror.

The brilliancy of many precious stones is due to their large refractive indices and therefore small critical angles. When light enters a cut diamond at any face it finds it very difficult to get out at most of the other faces, and hence bright beams issue at the one or two faces which are available for emergence.

54. The mirage. This is a phenomenon, on a large scale, due to total reflection from layers of air. In hot sandy deserts inverted images of distant objects and of the sky are often seen reflected as from a lake. By contact with the hot sand the lower layers of air become so heated that up to the height of a few feet the density increases upwards. Rays of light from a distant object entering these layers of rarefied air obliquely downwards become bent up more and more by refraction the lower they penetrate, and at last fall on a stratum at an angle greater than the critical angle (Fig. 63). Here total reflection takes place, and the rays becoming bent up more and more in traversing the denser layers above, at last reach the observer's eye as if they came from a point as far below the reflecting layer as the object is above it, while at the same time he sees the object direct by rays which do not pass down into the reflecting layer. Thus in the figure the appearance is that of a palm-tree standing by a tranquil pool of water.

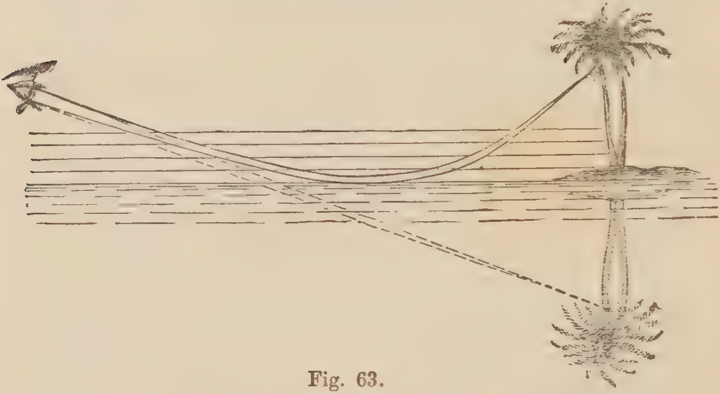

Fig. 63.

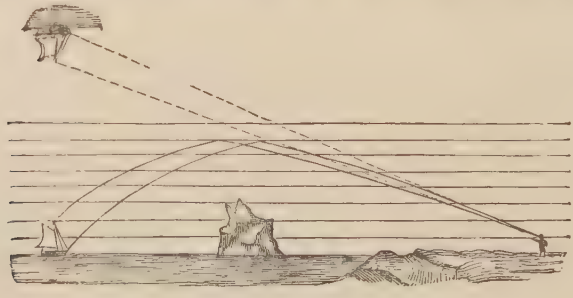

Fig. 64.

A similar mirage can often be seen across lakes on tranquil autumn mornings. In this case it is the water that heats the lower layers of air.

In the Arctic regions inverted images of ships and other objects are sometimes seen in the air, even though the objects themselves may be below the horizon. This is due to the very low temperature of the ice and sea cooling the lower atmospheric strata so much that their density increases rapidly downwards. Then rays passing obliquely upwards from the objects into the rarer layers become more and more bent down until they suffer a total reflection as in the other case, and as shown in Fig. 64.

55. Geometrical construction for critical angle. Let O be a point in the plane of separation A B (Fig. 65) of two media *a* and *b*. It is required to find the direction of a ray which, passing through the medium *b* and incident on A B at the point O, is just totally reflected.

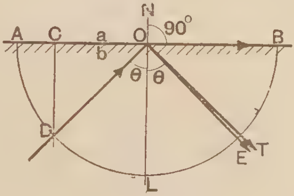

Fig. 65.

Draw N O L a normal at O, and on any convenient length A B as diameter describe a semicircle A L. From O A cut off a part O C equal in length to $OA/{}_a\mu_b$. Draw C D perpendicular to A B cutting the circle in D. Then D O is the direction of the critical incident ray, for it is evident that

$$\sin DOL = \sin ODC = \frac{OC}{OD} = \frac{OC}{OA} = \frac{1}{{}_a\mu_b} \quad \therefore \angle DOL$$

is equal to θ, the critical angle. The refracted ray O B skims the surface, the reflected ray O E of course makes an angle θ with O L. If the angle D O L exceed θ by ever so little, all the light is reflected taking the direction O T.

56. The glass block of Exp. 24 may also be used to determine the index of refraction, by an experiment in which the light is totally reflected from one face.

Exp. 29. Place the block upon a sheet of paper as before. With the eye in the neighbourhood of P_4 (Fig. 66) sight a pin placed at P_1 by observing its reflection in the face B C.

Now insert pins at P_2, P_3 so that P_1 P_2 P_3 P_4 appear in the same straight line. Remove the glass, and produce P_1 P_2 and P_4 P_3 to meet A B and D C at M and N. Between M and N the ray has been reflected at the surface B C. To find its path produce A B to M′, making B M′ equal to B M. Join N M′, cutting B C in Q. Then P_1 P_2 M Q N P_3 P_4 is the complete path of the ray from P_1 to the eye. Draw normals at M and N and from the values of i, r, i', r' or p_1, p_1', p_2, p_2' calculate two values of μ. They should agree very closely.

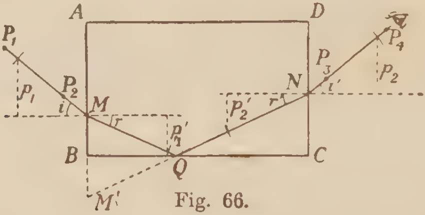

Fig. 66.

57. Refraction at a single plane surface. So far we have dealt only with the refraction of a single ray; we shall now consider the refraction of small pencils *directly** incident on the surface of separation of the media. Let *m m* (Fig. 67) represent the surface of separation of two media, *a* and *b*, of which *b* is the denser, and let A B represent one of the extreme rays of a diverging pencil of light directly incident

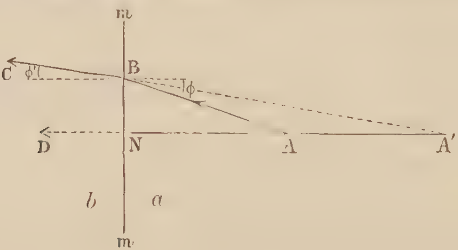

Fig. 67.

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* A pencil of light is *directly* incident on a surface when the axis of the pencil is perpendicular to that surface.

on *m m* along A N. A B is refracted at B along B C, while A N, being normal to *m m*, passes on along N D without suffering deviation. The focus of the refracted pencil will now be found at the point A′ from which B C and N D apparently diverge. It thus appears that A′ and A are conjugate foci, and that A′ may be considered as the image of A formed by refraction at the surface *m m*. It now remains to determine the relation between the distances of A and A′ from that surface. Let ϕ and ϕ' denote the angles of incidence and refraction, and μ the refractive index* for the case considered. Then, since ϕ and ϕ' are respectively equal to the angles B A N and B A′ N (Euc. i. 29), we have—

$$\mu = \frac{\sin\phi}{\sin\phi'} = \frac{\sin BAN}{\sin BA'N} = \frac{BN \,.\, BA'}{BA \,.\, BN} = \frac{BA'}{BA}.$$

But if B N is small—that is, if the incident pencil is small—then B A and B A′ are approximately equal to N A and N A′, and we have—

$$\mu = \frac{NA'}{NA}$$

Now, adopting the notation of Art. 39, and retaining the sign convention there explained, let N A be denoted by u and N A′ by v. Then—

$$\mu = \frac{v}{u}.$$

$$\therefore v = \mu u. \qquad (5.)$$

That is, the distance of the image from the plane refracting surface is μ times that of the object.

This explains why, on looking vertically downwards, the depth of a pond of water appears to be only three-fourths of what it really is. Let O (Fig. 68) represent an object at the bottom of the pond; then, after refraction at the surface of the water, the small direct pencil incident along

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* In what follows, μ always denotes the refractive index for refraction in the direction in which the light is supposed to be travelling.

O N appears to diverge from I—that is, the object O is seen at I. From relation (5) obtained above we get—

$$IN = \mu . ON.$$

Now μ from water to air $= \frac{3}{4}$.

$$\therefore IN = \tfrac{3}{4} ON.$$

In exactly the same way the apparent thickness of a plate of glass, or other transparent medium, as seen by an eye looking along a normal to the surface of the plate, is less than its actual thickness. For if O (Fig. 68) represent an object close to the face of the plate remote from the eye, then its apparent position is at I—that is, I N is the apparent thickness of a plate of actual thickness O N. Hence, if t denotes the thickness of the plate, its apparent thickness is given by μt, where μ is the index of refraction *from the medium into air.*

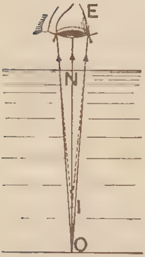

Fig. 68.

The result is true only in the case of small direct pencils.

Example. A small air bubble in a piece of glass with a plane surface is 3 inches below that surface; find its apparent distance from an eye looking at it, along a normal to the surface, from a point 8 inches from the surface. (Index of refraction from air to glass $\frac{3}{2}$.)

Here, applying $v = \mu u$, and remembering that the light is supposed to be travelling from glass to air, and that therefore $\mu = \frac{2}{3}$, we have—

$$v = \tfrac{2}{3} \times 3 = 2 \text{ inches.}$$

Therefore the apparent distance of the bubble from the eye $= 8 + 2 = 10$ inches.

Exp. 30.—To determine the refractive index of a solid (or liquid). Cut a thin slip of stamp-paper and stick it in a vertical position at O to the edge of the glass block A B (Fig. 69) already used in Exps. **24** and **29.** Place the block on the table and draw in the normal O N P. With the eye on this normal, look at O through the glass; observe that it appears nearer. It appears at I, and to locate I place a pin on the normal and with the eye in several positions not far distant from the normal adjust it until its image by reflection coincides with I. Note its final position, P. Remove the block, make I N equal to N P, then I is the position of the image of O and $ON/IN = \mu$.

If the refractive index of a liquid is required, the liquid must be placed in a glass cell, whose walls are thin, plane and parallel. The operations are then the same as for the solid block, the refraction of the glass walls being neglected.

It is also possible to work at the end of the block. Place the slip of paper, O, at the corner; I can then be located by a pin, which is moved along the end of the block. When the pin occupies the position of the image of O, it can be pressed into the paper.

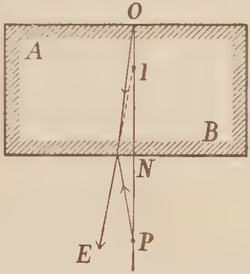

Fig. 69.

Exp. 31. A more accurate method is to use a low-power microscope which has a vertical adjustment and a fine scale by which the vertical motion can be measured. Cut a small star of paper, place it on the table and focus the microscope on it. Read the scale (1). Place the block on the paper-star. Elevate the microscope until it is again in focus. Again read the scale (2). Lay another small star of paper on the top of the block and again elevate the microscope until this is in focus. Again read the scale (3); the difference between (2) and (3) is equal to I N, and that between (1) and (3) is O N.

Example. A microscope was focussed on the bottom of a trough. Water was then placed in the trough and to again focus the bottom the microscope had to be raised 1 cm. The microscope was now focussed on a speck lying on the surface; in so doing it had to be raised 4 cms. above its first position. Find the refractive index of water.

Using Fig. 69, O N = 4 cms., I N = 3 cms., $\therefore \mu = \frac{4}{3} = 1{\cdot}3$.

Strictly, I is the conjugate focus of O only when the angle of the pencil diverging from O is infinitely small; as this angle increases, the focus I approaches nearer and nearer the surface (Fig. 70), until, when it is equal to twice the critical angle for the media, the point I coincides with N. If the angle of the pencil be greater than 2 θ (where θ denotes the critical angle), then all the rays making angles greater than θ with the normals at the points of incidence are totally reflected, and do not emerge from the water. Hence, remembering the reversibility of the direction of the passage of the light travelling along any path, it follows that an eye placed at O and looking upwards will see all external objects, most of them greatly distorted, comprised within a vertical cone whose semi-vertical angle is equal to the critical angle. The water surface outside this cone acts as a mirror, reflecting rays from objects lying below it.

If now we consider an oblique conical pencil O *a b c d* emanating from O and entering an eye placed at E, it does not diverge from a definite point after refraction. Two special cases may be considered. If the pencil be much narrower horizontally than vertically, it may be regarded as a number of rays all lying in (or close to) the vertical plane in which the figure is drawn. This will diverge after refraction from O′, almost exactly. If the pencil be wider horizontally than in plane of figure, it can be regarded as

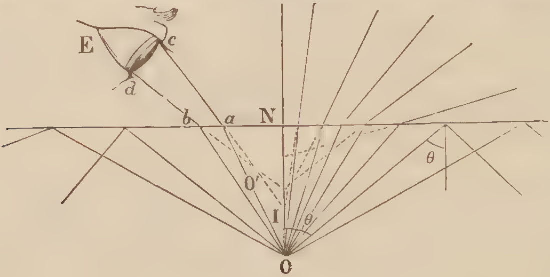

Fig. 70.

an aggregate of rays all equally inclined to O N, and these diverge after refraction from I. For other forms of pencil neither of these results is true; but the emergent pencil consists of rays, every one of which cuts (very nearly) two "*focal lines*," one at I in the plane of the figure, and one at O′ perpendicular to it.

If we are observing the point O with two eyes situated on a level and at the same distance from O N, these receive two narrow pencils which appear to come from the same point I *on the normal O N*. If we are observing with two eyes in the same vertical plane through O N, *i.e.* the plane of figure, the rays received appear to diverge from a point

*O′ not on the normal O N but on the observer's side of it.** If the eyes are held obliquely, the image seen is confused and cannot be located at a definite point.

Thus the apparent thickness of the medium becomes less and less as it is looked at more and more obliquely, and finally becomes zero when the direction of vision is parallel to its surface. This explains why the flat bottom of a vessel full of water appears slightly concave; the points vertically below the eye are seen by direct pencils, but the surrounding points by slightly oblique pencils, so that the water appears shallower as the range of vision travels outwards from the point vertically below the eye. If the eye be moved along parallel to the surface of the water, this appearance of concavity moves along with it, and thus an apparent wave motion is given to the bottom. For the same reason the depth of a pool of water appears to increase as we approach it and to diminish as we recede from it.

58. Image of a point seen by direct refraction through a plate. When a plate of glass or any transparent substance is interposed between the eyes and a near object, the distance of the eyes from the latter is apparently diminished.

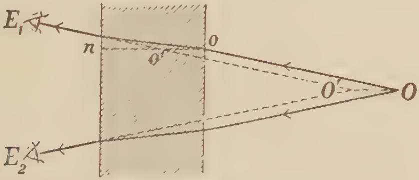

Fig. 71.

This is evidently due to the apparent diminution in the thickness of the plate; and, if t denotes the actual thickness, then the apparent thickness, t', is given by $t' = \mu t$, and the position of the object is apparently nearer the eye by a distance $(t - \mu t)$ or $t\,(1 - \mu)$, where μ denotes the index of refraction *from the plate to air*. Fig. 71 shows how this apparent change of position is effected. An object at O is seen at O′, the virtual focus of the refracted pencils which enter the eyes at E_1, E_2. O O′ represents the apparent change of position, and being equal to $o\,o'$ is equal to $o\,n - o'\,n$; that is, if O O′ be denoted by d, we have—

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* Fig. 70 well represents this case if *c* and *d* are taken to be positions of the eyes of the observer.

$$d = t\,(1 - \mu). \qquad (6.)$$

If the refractive index *from air to the plate* be used, the formula becomes—

$$d = t\,\frac{\mu - 1}{\mu}.$$

59. **Images produced by a plate with parallel faces.** Let O (Fig. 72) represent an object placed in front of the plate. Rays reach the nearest face of the plate in all directions from O. Consider the ray O a. It is partially

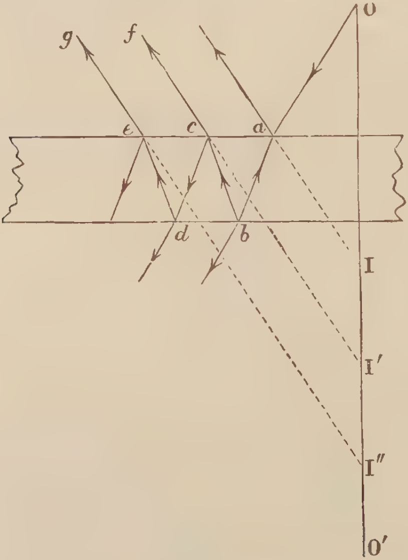

Fig. 72.

reflected from the first face at *a*, and an image due to this reflection is seen at I. But a portion of the light incident at *a* is refracted into the plate along *a b*, and, on incidence, at *b*, on the second face of the plate, a portion is reflected along *b c*, and the remainder refracted out into the air. The first portion, travelling along *b c*, again suffers partial reflection and refraction at *c*, and the emergent ray, *c f*, gives rise to another image I′, fainter than the first at I, because of the loss of light at *b* and *c*. Similarly, after reflection at *d*, and refraction at *e*, the light emergent along *e g* gives rise to another image I″ fainter than that at I′. In this way, by continued reflection and refraction, a series of images are formed approximately on the line I O′; each member of the series becoming fainter and fainter as the number of reflections by which it is produced is increased.

When we stand in front of a thick plate-glass mirror and examine our reflection in it, there is no apparent confusion, because the images formed by the two surfaces are almost exactly superposed, and still more because the second image (that due to the silver) quite overpowers by its brilliancy the feeble first image formed by the front surface of glass. But if we hold a finger, or better, a candle-flame, near the glass, and look obliquely at its reflection, we at once see both these images partly overlapping each other, and the second much brighter than the first. On looking more obliquely, the first image becomes brighter and the second less bright, and a third image, fainter than either appears. On looking still more obliquely, a fourth, and perhaps a fifth, image will be seen; and now the first image is the brightest, and the others show a gradual diminution of brightness.

60. Multiple reflections from thick mirrors.

Exp. 32. Hold a candle flame near a thick glass mirror and look obliquely at its reflection. Several images will be observed, partly overlapping each other, the second usually much brighter than the first, and the third, fourth, etc., gradually diminishing in brightness.

The explanation of their formation is as follows :—Let O and E (Fig. 73) represent object and eye; then all the rays (or, rather, the axes of the narrow diverging pencils) by which O is seen by E must on emergence from the plate converge to the eye. O emits light in all directions. Of the rays emitted one—*viz.* O a—is partly reflected at the front surface of the plate, and the reflected part passes to the eye along a E. The other part penetrates the glass; but, since none of this on emergence will enter E, we need consider it no further.

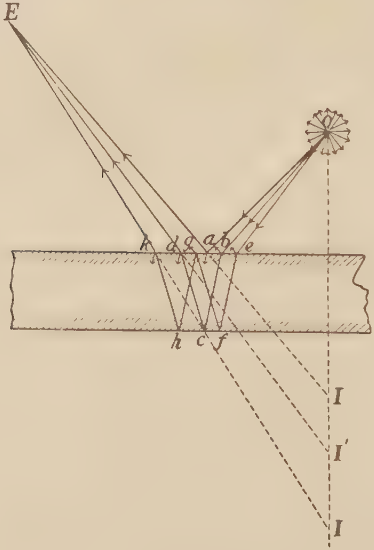

Fig. 73.

Another ray, O b, suffers one internal reflection at c, and enters the eye by the path O b c d E. Parts of O b are reflected at b and at d, but they can be neglected.

A third ray, O e, reaches the eye by means of the path O $e\,f\,g\,h\,k$ E. Portions of it reflected at e and k and emergent at g need not be considered. Other rays can be treated in a similar manner, and, if we consider each of these rays as the axial ray of a small conical pencil, it will be obvious that these pencils will, to an eye at E, be apparently coming from images I, I′, I″, which are on the normal through O, but which are not quite equidistant from each other.

The number of images seen depends upon the polish of the reflecting surface, for, after a certain number of reflections and refractions, the quantity of light reaching the eye becomes too small to excite the sensation of vision, and the loss of light by reflection at any surface depends upon the degree of polish of that surface. In performing this experiment it will be noticed that the first image increases in intensity as the angle at which it is seen is increased, and at very oblique incidence it becomes much the brightest. This shows that the quantity of light reflected from a glass surface increases as the angle of incidence increases.

When we stand in front of a thick plate-glass mirror and examine our image in it there is no apparent confusion, because the images formed by the two surfaces are almost exactly superposed, and still more because the second image (that due to the silver) quite overpowers by its brilliancy the feeble first image formed by the front surface of the glass.

Although in ordinary looking-glasses these multiple images are scarcely ever noticed, and are of no importance, in optical instruments they would be most inconvenient, and their formation is prevented by silvering the glass on the *front* surface and polishing the silver deposit as highly as possible. When great brilliancy is not necessary a very fair single-image reflector may be obtained by coating the back of a piece of plate glass with lampblack, which absorbs all the light except that going to form the first image.

If the luminous point O is at a great distance away, so that the rays O a, O b, O c, etc., may be considered parallel to each other, only one beam, and that of parallel rays, will enter the eye; and hence only one image is seen in the plate or mirror. If, however, the plate be not exactly uniform, and its faces not plane and parallel, more than one image of a distant point will be seen; and hence this experiment affords a severe test of the *goodness* of a plate.

61. Exercise.—**Construction of the refracted ray through a sphere or cylinder.** We shall assume that the ray is travelling in a diametral plane. Let O Q L (Fig. 74) be a diametral cross section of a homogeneous sphere or cylinder of material whose refractive index is μ, C the centre of the section, and A O an incident ray. Draw N O C L a normal through O, meeting the surface again in L, and produce A O to meet the surface again at P. The angle A O N is the angle of incidence. It is equal to the angle P O L. Join P L. Divide L P at R so that $\mu \,.\, \mathrm{L\,R} = \mathrm{L\,P}$ and with L as centre describe an arc cutting the surface at Q. Join O Q. Then O Q is the refracted ray within in the body. *Proof.* The angles O P L, O Q L being in a semicircle are right angles. Therefore

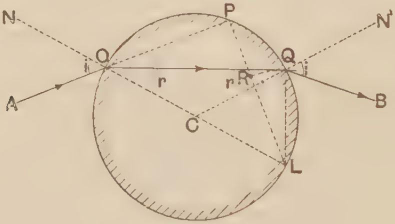

Fig. 74.

$$\sin i = \sin \mathrm{L\,O\,P} = \frac{\mathrm{L\,P}}{\mathrm{O\,L}} \text{ and } \sin \mathrm{L\,O\,Q} = \frac{\mathrm{L\,Q}}{\mathrm{O\,L}}.$$

Therefore

$$\frac{\sin i}{\sin \mathrm{L\,O\,Q}} = \frac{\mathrm{L\,P}}{\mathrm{O\,L}} \div \frac{\mathrm{L\,Q}}{\mathrm{O\,L}} = \frac{\mathrm{L\,P}}{\mathrm{L\,Q}} = \frac{\mathrm{L\,P}}{\mathrm{L\,R}} = \mu$$

(by construction), *i.e.* the ∠ L O Q is the angle of refraction. Denote it by r, then ∠ C Q O = r and therefore to get the emergent ray draw C Q N′ a normal at Q and make the angle N′ Q B equal to the angle A O N Note that the rays are symmetrical about a perpendicular drawn from C to O Q.

62. Opacity of mixtures of transparent substances. If some paraffin oil and water, or any other two liquids which will not mingle, and which have no chemical action on each other, be shaken in a test-tube the mixture becomes opaque like milk. The result of the agitation is to break up each liquid into a multitude of minute drops, each one of which retains all the transparency that the oil and the water in bulk possessed. But when a ray of light falls on the mixture, and encounters first a drop, let us say, of water, a certain proportion of the light will be reflected from the first surface of the drop, the rest passing through the drop until it encounters a neighbouring drop of oil. Here another reflection takes place, and the weakened ray passes through the drop of oil till it encounters a drop of water, when further reflection and further weakening takes place. Since there must be scores of such reflecting surfaces in every tenth of an inch of the mixture, it will be apparent that the light will be unable to penetrate directly to any considerable depth, and the *opacity* is at once explained; and the milky whiteness also, for the mixture reflects the light freely instead of allowing it to pass freely through it away from the eye. Foam is white and opaque for a similar reason—it being a mixture of minute particles of air and water, both of which are separately transparent. Milk also owes its whiteness to the same cause, for it consists of a multitude of minute globules of transparent fat floating in a transparent watery liquid. Snow and crushed glass are white and opaque for similar reasons. If two transparent liquids of precisely the same absolute index of refraction were shaken together, no such results would follow, for there would be no internal reflections at the bounding surfaces.

Again, if a colourless transparent solid were immersed in a colourless transparent liquid of the same refractive index, the solid would be invisible. An approach to this condition may be made by immersing a glass rod in glycerine, when it will be found that the existence of the rod might be easily overlooked on a casual glance. A fibre of cotton when seen under the microscope is nearly transparent, but paper, which is a feltwork of such fibres, is opaque, because the interstices between the fibres are occupied by air, which has a very different refractive index from the cotton. In the manufacture of tracing-paper the air is replaced by greasy substances, whose index of refraction approaches much more nearly to that of the cotton fibres than air does, consequently there is much less internal reflection and more transparency than in the untreated paper. The increased transparency of a linen lantern screen on wetting it is similarly explicable.

63. Atmospheric Refraction and its Effects. If the earth had no atmosphere the rays of light proceeding from a celestial body would travel in straight lines right up to the observer's eye or telescope, and we should see the body in its actual direction. But when a ray S *a* (Fig. 75) meets the uppermost layer A A′ of the earth's atmosphere, it is refracted or bent out of its course and its direction changed to *a b*. On passing into a denser stratum of air at B B′, it is further bent into the direction *b c*, and so on; thus, on reaching the observer the ray is travelling in a direction O T, different from its original direction, but in the same vertical plane.

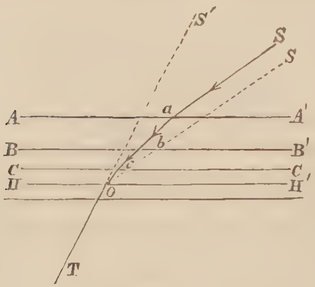

Fig. 75.

The body is therefore seen in the direction O S′, although its real direction is *a* S or O S. Also, since the successive horizontal layers of air A A′, B B′, C C′, . . . are of increasing density, the effect of refraction is to bend the ray towards the perpendicular to the surfaces of separation, that is, toward the vertical.

Hence *the apparent altitudes of the stars are increased by refraction.*

In reality, the density of the atmosphere increases *gradually* as we approach the earth, instead of changing abruptly at the planes A A′, B B′, C C′. . . . Consequently the ray, instead of describing the polygonal path S *a b c* O, describes a curved path, but the general effect is the same.

The elevation due to refraction increases as we go from the zenith to the horizon. At the latter place the elevation is about 33′; consequently a celestial body appears to rise or set when it is 33′ below the horizon. Thus the effect of refraction is to accelerate the time of rising, and to retard, by an equal amount, the time of setting of a celestial body. In particular the sun and moon, whose angular diameters are 32′ and 31′ respectively, appear to be just above the horizon when they are really just below.

When the sun or moon is near the horizon, it appears distorted into a somewhat oval shape. This effect is due to refraction. The whole disc is raised by refraction, but the refraction increases as the altitude diminishes, so that the lower limb is raised more than the upper limb, and the vertical diameter appears contracted. The horizontal diameter is unaffected by refraction, since its two extremities are simply raised. Hence the disc appears somewhat flattened or elliptical, instead of truly circular.

The apparent position of terrestrial objects also suffers from the same influence, distant bodies appearing elevated above their true position.

The duration of **twilight** (Art. 33) is also increased by refraction.

The twinkling of the stars. The air near the ground is more or less disturbed by convection currents, and thus the refractive index will vary from point to point even on the same level. The rays of light from a star, besides being bent as described above, will deviate now and then from their position in an atmosphere at rest, and so the light from a star will sometimes be concentrated at a point and sometimes decreased in intensity. Therefore, to a fixed observer, the star begins to twinkle or scintillate. That planets do not twinkle as much as stars is due to the angular size of the former; the directions of the rays from different parts of the planetary discs may vary very much, but the sum of the number of rays received by any given area—even if small—is very nearly constant, and thus uniform illumination results.

The above explanation is borne out by the fact that a star seen through a large telescope does not twinkle, the average amount of light falling on such a large area as an object glass being approximately constant.

EXERCISES V.

1. A piece of plate glass, 5 inches thick (refractive index 1·6), is placed between the eye and an object. Find what alteration will take place in the apparent distance of the object from the eye.

2. A ray of light passes from alcohol to a parallel plate of Iceland spar 1 in. thick, and then into air. The ray is incident on the Iceland spar at 45°. Make a scale drawing showing the exact path of the ray. The index of refraction of Iceland spar is $\frac{7}{4}$, and of alcohol $\frac{7}{5}$.

3. The sine of the critical angle for two media is $\frac{7}{9}$. What is the index of refraction from the rarer to the denser of the two?

4. Find the absolute refractive index of carbon disulphide, given that the relative index of refraction from carbon disulphide to glass is 0·9, and the absolute refractive index of glass is 1·512.

5. If a ray of light passes from one medium to a second, making the angle of incidence = 45°, and the angle of refraction equal to 30°, show that the refractive index for the media is $\sqrt{2}$.

6. The critical angle of a given medium is 60°. What is its refractive index?

Note.—When *the critical angle* or *the refractive index* of any medium is referred to simply, it must be understood that the other medium involved is vacuum.

7. A vessel, 6 inches deep, is filled with alcohol. What is the apparent depth of the liquid?

8. The refractive index of water is 1·33, and the velocity of light in air is 300,000,000 metres per second. Find the velocity of light in water.

9. In an experiment, similar to Exp. 24, the following readings were taken:—

Values of i,	10° 30′,	24° 24′,	39° 0′,	58° 0′.
Corresponding values of r,	6 48	15 48	25 45	34 12.

Find the mean value of μ.

10. In an experiment, similar to Exp. 29, the following readings were taken:

$$p_1 = 3\cdot05,\ p' = 2\cdot05. \quad p_2 = 2\cdot90,\ p_2' = 1\cdot94.$$

Find the mean value of μ.

11. Light incident at an angle of 60° on one face of an equilateral glass prism is deviated 30° at the first face. Draw a diagram showing the path of the rays through and out of the prism.

12. If you hold a glass of water with a spoon in it a little above the level of the eye, and look upwards at the under surface of the water, you will find that you are unable to see that part of the spoon which is above the water. Explain this.

13. A ray is incident on the plane surface of a glass block at an angle of 60°. The refractive index of the glass is $\sqrt{3}$. Find (1) by drawing, (2) by calculation, the angle of refraction.

14. A thick plate of glass is interposed obliquely between a candle and the observer's eye. Will the apparent position of the candle be altered by the glass? Draw a diagram illustrating the answer.

15. You have a piece of thick plate glass through which you look at a vertical pole (say a telegraph post). The glass being held so that part of the pole is seen directly, and part through the glass, describe and explain the change in the apparent position of the part of the pole which is seen through the glass when the latter is turned about a vertical axis.

16. A cube of clear glass is placed on a horizontal sheet of white paper so that the middle of its lowest face is over a black spot on the paper. Draw and explain diagrams to illustrate the apparent position of the spot to an observer looking at it through the top.

17. A hollow watertight prism containing air, with flat glass sides, is immersed in a glass tank full of water. Draw and explain a diagram showing the path of a ray of light passing through the water and the prism.

18. A candle is placed at a given small distance in front of an ordinary looking-glass made of thick plate glass quicksilvered on the back, and a person looking obliquely into the mirror sees several images of the candle. Explain this, and draw a careful diagram to illustrate it.

19. In the evening take a thick mirror and look obliquely in it at the reflection of a bright star or planet, say Jupiter. The appearance is that of a star or planet attended by several moons. Explain the illusion.

20. Explain the quivery appearance seen above hot rocks or bricks, and the streaky appearance of water in which ice, sugar, or acid is being dissolved.

21. A piece of a colourless mineral is dropped into a colourless liquid; the mineral is invisible in the liquid. How are the refractive indices of the liquid and of the mineral related?

22. A beam of light issues from a bright point 3 feet above the surface of still water, and falling obliquely on the surface is divided into two parts, one of which is reflected and the other refracted. Find the position of the point of incidence and its distance from the bright point, so that the reflected and refracted rays may be at right angles to each other.

23. Light from a bright point 6 inches above the surface of still water is reflected from the bottom of the vessel, which is 2 feet deep, as well as from the surface of the water. Show how to find the positions of the images formed by the reflections.

EXAMINATION QUESTIONS.

1. Explain in non-mathematical language, and by aid of a drawing, what is meant by the index of refraction of a transparent medium.

2. State the laws of the refraction of light by such substances as water or glass, and describe and explain experiments by which they can be demonstrated.

3. Explain why a straight stick partly immersed in water in an oblique position appears bent at the surface of the water.

4. Explain how to find the real depth of a pond from its apparent depth as seen by an observer vertically above the pond.

5. Enunciate by aid of a sketch, and in two statements, the law of refraction when light passes from a rarer to a denser medium. Also point out what is meant by the critical angle.

6. Prove that the apparent depth of a luminous object beneath a surface of water is only three-fourths of its real depth.

7. A ray of light passes from air into glass, the refractive index of glass with regard to air being 1·5. Given the angle of incidence at the common surface, draw a diagram to show how the angle of refraction may be accurately determined.

8. If the refractive index of a ray of light in passing from air to water be $\frac{4}{3}$, and in passing from air to glass $\frac{3}{2}$, find by aid of a diagram what it will be for the ray when passing from water to glass.

9. Light proceeds from a point at the bottom of a lake. Make a careful drawing of the pencil of rays after emergence from the water, and find the geometrical focus of the pencil.

10. Describe what a fish would see on looking towards the surface of the water in directions differently inclined to the horizon, and illustrate your descriptions by a diagram.

11. How would you experimentally verify the laws of refraction? What condition is necessary in order that a ray of light may be able to emerge from the plane surface of a refracting medium.

12. Walking by the side of a shallow stream of clear water of uniform depth, the gravelled bottom appeared to possess a wave-motion, the trough of the wave being always vertically beneath the observer. Explain this by means of a diagram.

13. The refractive index of water with respect to air is $\frac{4}{3}$, and the refractive index of water with respect to oil of turpentine is $\frac{9}{10}$. Show how to find the refractive index of oil of turpentine with respect to air.

14. Describe some experiment by which the phenomenon of total internal reflection may be produced and observed. State also how the minimum angle of incidence at which total internal reflection takes place may be determined.

15. Prove the formula $v=\mu u$, connecting the distances of two conjugate foci from the plane surface separating two different refracting media.

16. Draw to scale a figure showing the apparent positions of an object 3 feet beneath the surface of water to an eye receiving pencils of rays at different inclinations to the surface.

17. Explain, by help of a figure, the change produced in the apparent position of an object by interposing a plate of glass between it and the eye.

18. A fish swims in a glass tank ; a person whose eye is above the level of the water seems to see two fish. Draw a diagram to illustrate this, and give any explanation you think necessary.

19. A boy wades in a pond which everywhere reaches to about the level of his knees. On account of the water, some of the pebbles with which the bottom is covered are invisible, and others are not seen in their true places. Explain this, and illustrate your answer by a diagram.

20. A person arranges a basin of water and a candle in such a way that the reflection of the candle on the water seems to him to be in the same straight line as a coin at the bottom of the water. Draw a diagram to show the necessary arrangement, and give any explanation that you think necessary.

21. Explain by means of a diagram the fact that a fish in a river does not appear to a person upon the bank to lie in the direction in which it actually is. Show also the direction in which the person would appear as seen by the fish.

22. Explain clearly what is meant by saying that the refractive index of glass is $\frac{3}{2}$. Is the refractive index the same for all kinds of light? Describe any experiment by which you could prove the correctness of your answer.

23. Two pins A and B are stuck into a sheet of paper on either side of a rectangular block of glass resting on it, the pins being at some distance away from the block. Show how to trace experimentally the path of a ray through the block from A to B, and how to determine the refractive index of the block from the result.

The positions of A and B being fixed, but that of the block being variable, describe how you would place the latter to avoid getting an inaccurate result for the refractive index.

24. Describe what is meant by total internal reflection. Under what circumstances can it occur?

A stone is placed on the bottom of a large vessel filled with water. The bottom of the vessel is transparent. On looking through the bottom the stone can be seen by total internal reflection at the upper surface of the water, if the eye is some distance from the stone, but not if it is close to. Explain this.

25. A plate of parallel glass 3 cms. thick is placed between your eye and an object. Draw carefully a diagram to show the paths of the rays by which you see the image of the object. If the plane of the plate be perpendicular to the line drawn from the object to the centre of your eye, and if the refractive index of the plate be 1·57, calculate approximately how much nearer you the image is than the object.

26. An eye looks along a fixed line at a divided scale, through a thick glass plate with parallel faces. Show, by carefully drawn figures, that, as the plate is tilted, different points on the scale will be seen along the fixed line.

27. A piece of thick plate glass is held upright between the eye and a thin vertical rod, so that a portion of the rod is seen through the glass and a portion over the glass. At first the glass is perpendicular to the plane containing the rod and the eye. Describe the change that takes place in the appearance of the rod when the glass is turned slowly about a vertical axis until its faces are almost parallel to the plane containing the rod and the eye. Give careful diagrams explaining what is observed in the initial, final, and one intermediate position of the glass.

28. Enunciate the laws of refraction of light.

Draw three parallel straight lines, an inch apart, in the plane of the paper to represent rays of light incident upon a glass sphere of radius 2 inches, with its centre upon the last of the series, and trace, by a geometrical construction, the paths of the refracted rays within and beyond the sphere, taking the index of refraction of light passing from air to glass as $\frac{3}{2}$.

Taking your diagram to be correct for red light, indicate roughly how it would be altered if the incident rays were blue.

29. A ray of light is incident at 60° to the normal upon a polished glass surface. The refracted ray makes an angle of 90° with the reflected ray. Find graphically or by calculation the refractive index of the glass.

CHAPTER VII.

REFRACTION THROUGH PRISMS AND LENSES.

64. In this chapter we shall not consider dispersion, and must therefore be understood to deal with the refraction of rays and pencils of light *of definite wave-length*, and therefore of definite refractive index and *colour*. Such light is sometimes referred to as *monochromatic* or *homogeneous* light, and is conveniently obtained, of a yellow colour, from a flame coloured by the presence of a salt of sodium (see Exp. 48).

PRISMS.

65. Prisms. From an optical point of view, **a prism** is any portion of a medium lying between two plane faces inclined to each other at any angle. The line of intersection of these faces is known as the *edge* of the prism, and a section of the prism at any point in its length, perpendicular to this edge, is called a *principal section.* The **refracting angle** of the prism is the angle between its faces, as measured by the corresponding plane angle of the principal section. The prisms generally used for experiments are triangular prisms, in the geometrical sense of the term. The principal sections of such prisms are equilateral, isosceles, or scalene triangles, according to the purpose for which the prism is intended. When the section is equilateral the angle at each edge is equal to 60°, and thus there is no gain in having three edges; with an isosceles section there are two different angles available, and with a scalene section the angle at each edge is different, and thus the prism is equivalent to three prisms considered in the optical sense.

Exp. 33. Take a glass prism and look through it at any object. Observe that the object is apparently displaced towards the edge of the prism, and that its edges are coloured.

66. **Refraction through a prism.** In dealing with refraction through a prism, we shall consider only the case where the plane of incidence and refraction is coincident with a principal section of the prism.

Let A B C (Fig. 76) represent the principal section of a prism, and B A C the refracting angle considered; then, if the material of the prism be of higher refractive power than the external medium, a ray R N incident on the face A C, at N, is bent towards the normal on entering the prism, and, taking the course N N′, is incident on the face A B at N′, where it is bent away from the normal, and leaves the prism by the path N′ R′.

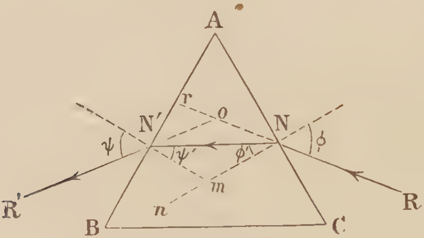

Fig. 76.

The ray R N is thus, after refraction through the prism, *deviated* from its original direction R N, and finally travels along N′ R′. The deviation of the ray is evidently measured by the angle *r o* N′; its *magnitude* is found to depend on the path of the ray through the prism, but its *direction* is always away from the refracting edge. There is one position for which this deviation is a minimum; when the prism is so placed that the incident and emergent rays make equal angles with the normals at their respective faces, then the deviation is a minimum, and the prism is said to be in the position of **minimum deviation**. This can be proved theoretically; but as the proof is beyond the scope of this work, we must, for the present, consider it as a fact established by experiment. (See Exp. **34**.)

The minimum deviation produced by any prism depends on the angle of the prism and the refractive index of its material relative to the external medium. We shall now proceed to establish an important relation between these quantities. In Fig. 76 let ϕ and ϕ' denote the angles of incidence and refraction at N, and ψ' and ψ the corresponding angles at N′.*

Then, if D denote the deviation produced, we have—

$$\mathrm{D} = r\,o\,\mathrm{N}' = o\,\mathrm{N}\,\mathrm{N}' + o\,\mathrm{N}'\,\mathrm{N} \qquad \text{(Euc. i. 32.)}$$

$$= (\phi - \phi') + (\psi - \psi').$$

$$\therefore\ \mathrm{D} = \phi + \psi - (\phi' + \psi'). \qquad (1.)$$

But, since the angle contained between any two lines is equal to that contained by lines perpendicular to them, we have, if A denote the angle of the prism—

$$n\,m\,\mathrm{N}' = \mathrm{B\,A\,C} = \mathrm{A}.$$

$$\text{But } n\,m\,\mathrm{N}' = \phi' + \psi'. \qquad \text{(Euc. i. 32.)}$$

$$\therefore\ \mathrm{A} = (\phi' + \psi'). \qquad (2.)$$

Substituting this value of $(\phi' + \psi')$ in (1), we get—

$$\mathrm{D} = \phi + \psi - \mathrm{A}. \qquad (3.)$$

Now, when the prism is in the position of minimum deviation, the ray passes *symmetrically* through the prism, and we must therefore have $\phi = \psi$ and $\phi' = \psi'$.

Therefore, from (3)—

$$\mathrm{D} = 2\,\phi - \mathrm{A}. \qquad \therefore\ \phi = \frac{\mathrm{D} + \mathrm{A}}{2}. \qquad (4.)$$

And from (2)—

$$2\,\phi' = \mathrm{A}. \qquad \therefore\ \phi' = \frac{\mathrm{A}}{2}. \qquad (5.)$$

But, if μ denote the refractive index of the material of the prism, relative to the external medium, then—

$$\mu = \frac{\sin\phi}{\sin\phi'}.$$

---

* At N′ the angle N N′ m is the angle of incidence; but, for the sake of symmetry with ϕ', it is here denoted by ψ' and not by ψ.

Therefore, substituting from (4) and (5), we have—

$$\mu = \frac{\sin \frac{1}{2} (D + A)}{\sin \frac{1}{2} A}. \qquad (1.)$$

This result, in connection with refraction through a prism in the position of minimum deviation, is of great practical importance.

Example. The refracting angle of a prism is 60°, and the minimum deviation produced in a pencil of monochromatic light is 40°. Find the refractive index of the prism, given that sin 50° = ·766

Here, applying—

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A},$$

we get—

$$\mu = \frac{\sin \frac{1}{2} (60 + 40)}{\sin \frac{1}{2} (60)} = \frac{\sin 50}{\sin 30} = \frac{\cdot 766}{\frac{1}{2}} = 1{\cdot}53.$$

When the angle of the prism is small, a very convenient expression for D may be obtained from the formula just established. Thus we have—

$$\mu = \frac{\sin \frac{1}{2} (D + A)}{\sin \frac{1}{2} A}.$$

Now, if D and A be so small that the angles $\frac{1}{2}$ (D + A) and $\frac{1}{2}$ A may be considered as approximately equal to the sines of these angles, we have—

$$\mu = \frac{D + A}{A}, \text{ or, } D = (\mu - 1) A. \qquad (2.)$$

Example. The minimum deviation of a ray by a prism of refracting angle 5° is 2°. Find μ.

$$\mu = \frac{D}{A} + 1 = \frac{2}{5} + 1 = 1{\cdot}4.$$

67. Conjugate foci in the case of refraction through a prism in the position of minimum deviation. It is a general law that, when any quantity is passing through its maximum or minimum value, a small change in the variable concerned produces very little effect on the magnitude of the quantity itself. For example, the magnitude of the deviation produced by refraction through a prism depends upon the path of the rays; but when the prism is in the position of minimum deviation, any small change in the path produces but little change in the magnitude of the deviation. Hence, for rays passing through a prism, by paths near to that of minimum deviation, the deviation which each undergoes is practically the same, and very nearly equal to the minimum value.

Hence, if a *small* pencil of rays coming from F (Fig. 77) be incident on a prism at such an angle that the axis passes along the path of minimum deviation, then all the rays will be deviated to an approximately equal extent, and will therefore, on emergence, be inclined to one another at nearly the same angle as before incidence. Hence, if produced backwards, the rays of the pencil appear to come from a point F′ such that F′O = FO. Similarly, if we imagine the path of the pencil to be reversed, we see that a convergent pencil having its focus at F′ would, after refraction through the prism, converge to the point F.

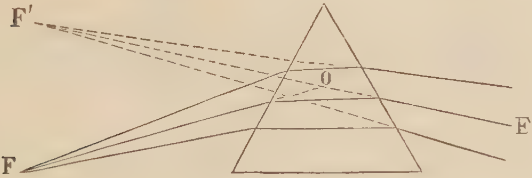

Fig. 77.

F and F′ are thus conjugate foci; and, if the pencil be incident near the refracting edge of a prism of small angle, placed in the position of minimum deviation for the axis of the pencil, we may neglect the thickness of the prism, and state that conjugate foci are on the same side of the prism and equidistant from its edge.

It follows from what has been said above that, if an object be placed at F, its image is seen at F′ by an eye placed at E. This image is evidently virtual and displaced from the position of the object towards the edge of the prism (Fig. 78).

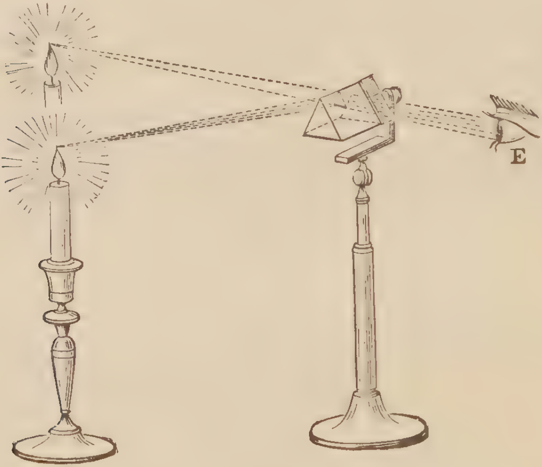

Fig. 78.

68. Practical illustration of minimum deviation. **Exp. 34.** Take a prism, P (Fig. 79), and cut out a circular piece of cardboard, C, so that when the prism is mounted on it its edges stand vertically over the circumference of the disc. Stick P to C by means of soft wax. Fasten a piece of cartridge (drawing) paper to a board and in the middle of the sheet describe a circle of the same size as C. Rule a straight line B D across the paper and erect two pins at B and D. Place P and C in position, one face of P being nearly normal to B D. Look through the prism from the other side and erect two pins E_1 and F_1 in the paper on the same side of the eye, so that E_1, F_1, and the images of B and D appear in a straight line. The images of B and D will appear a trifle indistinct and coloured, and the alignment must be effected by means of their centres. Test the observation by placing the eye beyond B and looking along B D E_1 F_1. Mark the position of A on the paper with a pencil point 1, remove E_1 F_1, and rule in the line E_1 F_1. Still keeping C exactly over the pencilled circle, turn it around so that A moves a little to the right. Repeat the observations. Denote the new position of A, E_1 and F_1 by 2, E_2 and F_2 respectively. Repeat for several positions of A until the angle of incidence is large and the images of the pins very indistinct. It will be found that as P is turned around, the line of pins on the eye side of the prism moves up to a position E F and then retreats. Remove the prism and card. Mark in the position of the prism which gave E F, produce B D, F E to meet the prism faces and show that the angle of incidence is equal to the angle of emergence. Now measure the angle between the prolongation of BD and EF. It is equal to the D of Art. **66**. Measure the angle A of the prism by a protractor and find the refractive index of the material composing the prism by means of the formula

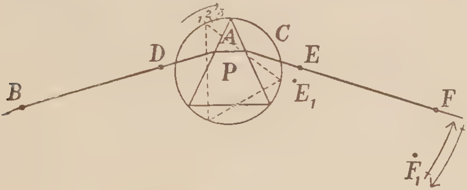

Fig. 79.

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}}.$$

69. Assuming that experiment proves that there is only one position of the prism with regard to the incident beam which makes the deviation a minimum, it is easy to show theoretically that this position occurs when the incident

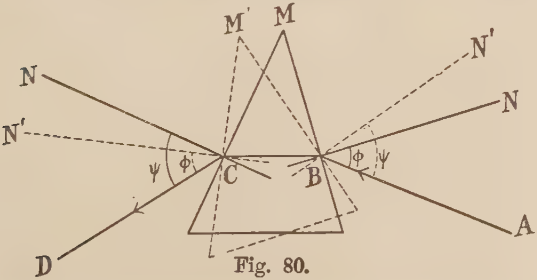

Fig. 80.

and emergent rays occupy symmetrical positions with respect to the prism. For if the incident beam A B (Fig. 80) is fixed in direction, and the position of the prism M is such as to make the deviation of the ray A B C D a minimum when the angles of incidence and emergence are ϕ and ψ respectively, ϕ and ψ being unequal to each other, then also will the deviation be a minimum when the angles of incidence are respectively ψ and ϕ. So that there are two positions of the prism, namely, M and M′, which render the deviation a minimum. **Exp. 34** shows this to be false, and it will be noted that the two cases only merge into one when ϕ and ψ are equal to each other.

Lenses.

70. Lenses. A lens may be generally defined as a portion of a medium enclosed between two surfaces of definite geometrical form and having a common normal. Usually these surfaces are portions of spheres or plane surfaces, and the medium most generally employed is glass. Lenses of this form may be considered as solids of revolution. For example, if any one of the sections shown in Figs. 81 and 82 be supposed to revolve round a central horizontal axis in the plane of the paper, the solid described by such revolution determines the form of the lens corresponding to that section.

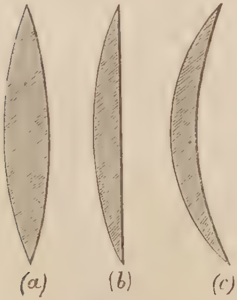

Fig. 81.

It is usual to divide lenses into two classes:—

1. **Convex lenses** (Fig. 81). Of these there are three chief forms—

(*a*) **Double convex.**

(*b*) **Plano-convex.**

(*c*) **Concavo-convex** [*converging meniscus*].

The distinguishing characteristic of these lenses is that they are *thicker at the centre than at the edges.*

2. **Concave** lenses (Fig. 82). Corresponding to the three forms of convex lenses we have—

(*a*) Double **concave**.

(*b*) Plano-**concave**.

(*c*) Convexo-**concave** [*diverging meniscus*].

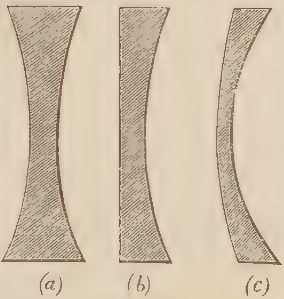
(a) (b) (c)

Fig. 82.

The distinguishing characteristic of this class is that the lenses *are thinner at the centre than at the edges.*

The action of any of these forms of lenses on a pencil of rays passing through them depends on the refractive index of the medium of which they are made, relative to the surrounding medium. Usually we have to deal with glass lenses surrounded by air, that is, the medium of the lens is of higher refractive power than the surrounding medium. In this case, **convex** lenses cause the rays of a pencil to become more convergent, or less divergent after passing through them, and for this reason are sometimes called **converging** lenses. Similarly, **concave** lenses are called **diverging** lenses because the rays of a pencil are always more divergent or less convergent after refraction through them than before.*

This action of convex and concave lenses may be explained in the following way. The section of a double convex lens may be considered as rather similar to that of two prisms placed base to base as in Fig. 83. Consider the rays P A and P B incident on the prisms at A and B. As explained in Art. **66**, these rays are deviated away from the edges of the prisms on which they are incident, and are thus less divergent after refraction. The path of the rays P A and P B after passing through the lens, depends on the magnitude of the deviation produced; they may either diverge from P′, run parallel, or, if the deviation be sufficiently great, converge to a point, P″.

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* When the refractive index of the substance of the lens is less than that of the surrounding medium, then a *convex* lens acts as a *diverging* lens, and a *concave* lens as a *converging* lens.

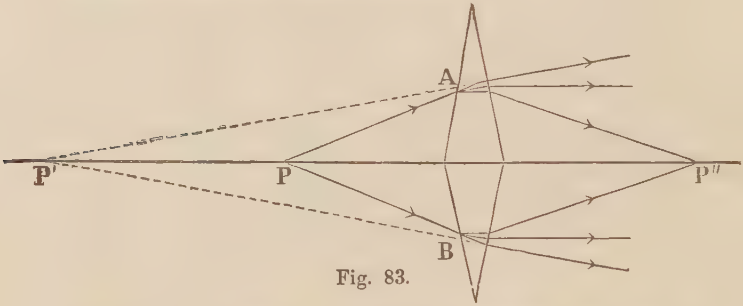

Fig. 83.

Similarly, the section of a double concave lens may be considered as rather similar to that of the two prisms placed apex to apex, as in Fig. 84. In this case the rays P A and P B are refracted away from the edges of the prisms, that is, from the centre of the lens, and, after refraction, appear to diverge from the point P′; the rays are thus more divergent after passing through the lens than before.

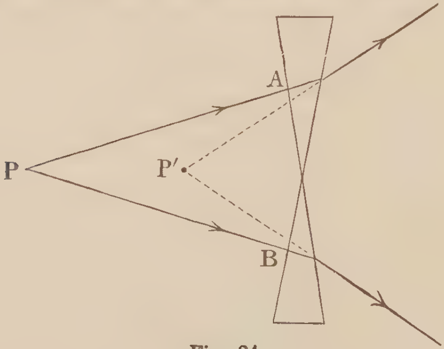

Fig. 84.

In the case of the prisms shown in Figs. 83 and 84, the positions of P′ and P″ will depend on the positions of A and B, but in the case of a lens, owing to the curvature of the surface, all rays coming from P would, after refraction, pass through the same point. When this is accurately the case the curvatures of the surfaces of the lens are specially adapted to the existing conditions, and the lens is said to be *aplanatic*; but, for ordinary lenses, with spherical or plane surfaces, this is only approximately the case, and the defect resulting from this want of accuracy is known as *spherical aberration.** (See Art. **83**.)

A lens, being a solid of revolution, is symmetrical about its centre, and hence all sections passing through the axis of revolution are similar. It thus follows that what has been explained above, for one section, is true for all similar sections, and consequently, if a *pencil* of light, diverging from P, be refracted through a lens, all the rays are symmetrically deviated, and, after refraction, pass through the same point.

71. Influence of curvature of surfaces of lens on deviation. Consider the refraction of the rays P *a* *b* P′

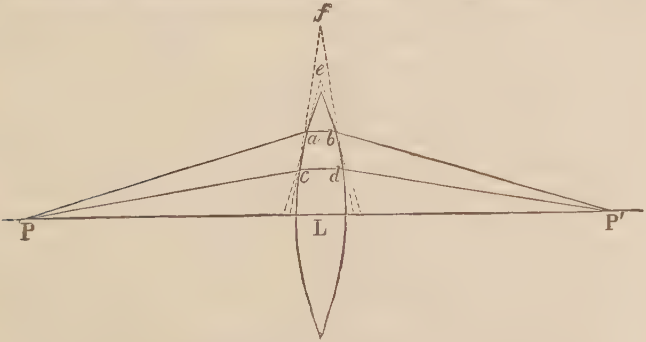

Fig 85.

and P *c* *d* P′ through the lens L (Fig. 85). It is evident from the figure that, in order that the rays may pass through P′, the deviation of P *a b* P′ must be greater than that of P *c d* P′. At *a* and *b* draw tangent planes to the surfaces of the lens meeting in *e*, and at *c* and *d* draw tangent planes meeting in *f*. Now the deviation in the case of the ray P *a b* P′ is that due to the prism of refracting angle *a e b*, and the deviation for the ray P *c d* P′ is that due to a prism of angle *c f d*. But, when the angle of the prism is small, the deviation produced is approximately proportional to the angle of the prism (Art. 66). Therefore, in this case the deviation for the ray P *a b* P′ is greater than that for P *c d* P′, and thus it is possible for both rays to pass through P′.

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* When the surfaces of the lens are only very small portions of spherical surfaces, spherical aberration is almost negligible, and the lens is, for all practical purposes, *aplanatic*.

72. Definitions. The *principal axis* of a lens coincides with its axis of revolution, and, when the surfaces of the lens are spherical, passes through the centres of curvature of these surfaces. When one surface is plane, and the other spherical, the axis passes through the centre of curvature of the spherical surface and is normal to the plane surface. **The optical centre of a lens** *is that point, on the principal axis, through which pass all rays, or, in certain cases, the prolongations of the portion of such rays which is within the lens, having their paths parallel before and after refraction through the lens* (O, Fig. 88).

It can be proved that the optical centre divides the thickness of the lens into segments proportional to the radii of curvature of the adjacent faces. In the case of double convex and double concave lenses the optical centre lies in the interior of the lens; in plano-convex and plano-concave lenses it is situated on the spherical surfaces, and in a converging or diverging meniscus it lies outside the lens on the same side as the surface of lesser radius of curvature.

Although the incident and emergent parts of a ray which passes through the optical centre are parallel, they are not in the same straight line; but, if the thickness of the lens be small, the displacement produced is negligible, and it may be stated that all rays passing through the optical centre of a thin lens suffer no deviation, but continue their course in the same straight line.

Any line, other than the principal axis, passing through the optical centre is called a *secondary axis.*

When a parallel pencil of light is incident on a lens in a direction parallel to the principal axis of the lens, the rays, after refraction through the lens, converge to or diverge from a point on the principal axis. This point is the (first) **principal focus** of the lens, and its distance from the optical centre of the lens, measured along the principal axis, is the **focal length** of the lens.

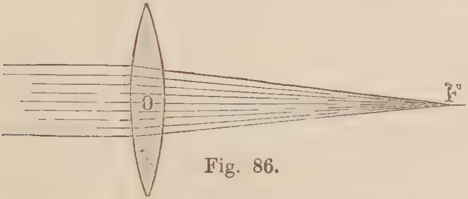

Fig. 86.

In the case of a convex lens, of any form, the parallel pencil of rays is made to *converge* to a point F (Fig. 86) on the other side of the lens. A concave lens (Fig. 87) causes the rays to *diverge* from a point F on the same side of the lens as the incident pencil. In both cases O F represents the focal length, and, applying the usual convention of sign, it is evident from Figs. 86 and 87 that, if distances be measured from O, the focal length of a *concave* lens is *positive* and that of a *convex* lens *negative.*

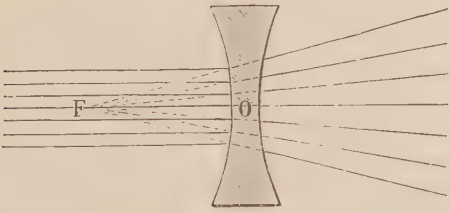

Fig. 87.

If the pencil of parallel light is incident in a direction parallel to a secondary axis A F_1, inclined at a *small* angle to the principal axis (Fig. 88), the focus of the refracted pencil is on the secondary axis at a point F_1, such that O F_1 is approximately equal to the focal length of the lens.

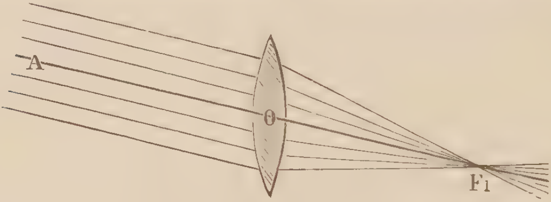

Fig. 88.

Exp. 35. Go into the sunshine and hold a convex lens in front of a piece of paper so that the lens is broadside on to the rays. Start with the lens up close to the paper and note the circle of light on the paper. It is almost as large as the lens. Now gradually move the lens away from the paper. Note that the circle of light on the paper gets smaller and smaller until it becomes almost a point (as in Fig. 86). The lens is now at a distance from the paper equal to its focal length, and the bright point is at the principal focus of the lens. If the experiment is done on a hot summer's day, note that the piece of paper at the focus is charred. (The experiment may be repeated, using the hand instead of the paper. The rise of temperature is then more evident.) Remove the lens still further from the paper : the spot of light now broadens out, showing that all the rays of light crossed at the principal focus. Repeat with a concave lens. No concentration of light occurs, the circle of light is always bigger than the lens (see Fig. 87). To see this circle it will be necessary to mount the lens over a hole in a piece of brown paper or cardboard to block out extraneous light.

Exp. 36. (To be done in a dark room.) Light a candle or a gas flame and place it in front of a small hole pierced through a screen. Mount a convex lens in a support and place it at a distance from the hole equal to its focal length. Now hold a second screen at various distances from the lens and observe that the circular patch of light on this screen is always of constant size, thus showing that the rays are leaving the lens in a parallel beam (see Fig. 94).

In the following articles we shall deal only with thin lenses.

73. Path of a ray through a lens. Let C and C′ (Fig. 89) denote the centres of curvature of the faces of the lens A B, and let the ray P A be incident on the surface of the lens at A. Join C A and produce it to N; then C A N is the normal at A, and the ray P A is refracted into the lens along A B, making an angle B A C with the normal such that—

$$\frac{\sin \mathrm{P\,A\,N}}{\sin \mathrm{B\,A\,C}} = \mu$$

where μ denotes the refractive index of the material of the lens relative to the surrounding medium. Similarly at B, the ray is incident on the second face of the lens and is refracted along B P′ in such a direction that

$$\frac{\sin \mathrm{P'\,B\,N'}}{\sin \mathrm{C'\,B\,A}} = \mu.$$

To determine, by this construction, the path of any given ray would be a very troublesome process; and it is therefore important to notice two particular cases in which the path is readily determined.

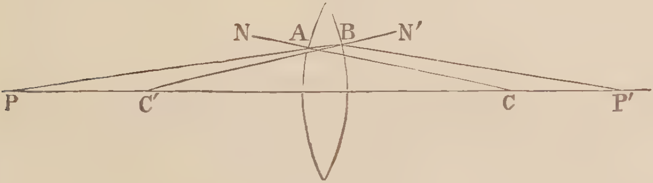

Fig. 89.

1. Any ray passing through the optical centre* of a lens continues its course in the same straight line (Art. 72).

2. When the incident ray is parallel to the principal axis, the refracted ray passes through the principal focus (Art. 72) if the lens is convex, or travels directly away from it if the lens is concave.

---

* For ordinary purposes the optical centre of a *thin* lens may be taken at any point in its thickness, on the principal axis.

74. Conjugate foci.* When rays of light, diverging from a point P (Fig. 90) on the principal axis of a lens, are refracted through the lens, the focus of the refracted pencil is another point P′, also on the principal axis. These points, P and P′, are called *conjugate foci*. When the point P is on any secondary axis *inclined at a small angle to th principal axis*, the point P′ is also on that secondary axis but it is important to notice that secondary axes have not the same relation to lenses as they have to mirrors. In the case of mirrors, the secondary axes have exactly the same geometrical relation to the spherical reflecting surface as the principal axis, but for lenses this is not the case, and refraction along secondary axes involves several complications which we cannot now consider. When, however, the angle between a secondary axis and the principal axis is *small* the laws applicable to refraction along the principal axis may be applied with approximately correct results.

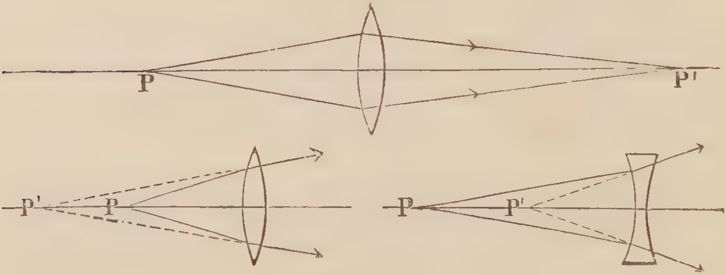

Fig. 90.

The relation between the distances of *conjugate foci* from the centre of the lens and the focal length of the lens will now be deduced by a simple geometrical method.

---

* It should be noticed that if the conjugate foci are both *real* the image of an object placed at either focus is formed at the other; but if one of the foci is *virtual*, then the image of an object placed at that focus is not formed at the other, but rays converging to the *virtual* focus are refracted through the conjugate focus. That is, the optical relation between conjugate foci assumes reversal of the direction of the light.

75. Relation between the distances of conjugate foci from the lens and the focal length. The proof of the following relation depends upon two facts: (1) That a ray directed to the optical centre of the lens passes through it undeviated. (2) That a ray initially parallel to the axis passes after refraction through the principal focus if the lens is convex, and travels directly away from the principal focus if the lens is concave.

Case of a concave lens. Let L (Fig. 91) represent a concave lens with centre at O, axis along X Y, and principal focus at F. Let A be a luminous point, not on the axis of the lens. To find the image of A, take a ray A *a* parallel to the axis of the lens. After refraction it takes the path *a c*, F *a c* being in a straight line. Also take a ray AO through the optical centre. It passes undeviated on to *d*. The rays *a c*, O *d* apparently diverge from A′, which is therefore the conjugate focus to A, and is the virtual image of A.

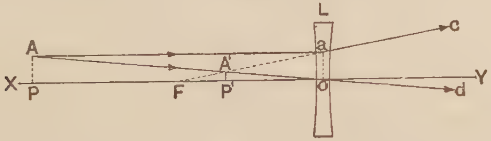

Fig. 91.

Draw A P, A′ P′ perpendicular to X Y. Then from the similar triangles A P O, A′ P′ O we have

$$\frac{AP}{A'P'} = \frac{OP}{OP'}.$$

Also from the similar triangles F *a* O, F A′ P′ we have

$$\frac{aO}{A'P'} = \frac{OF}{FP'}.$$

But A P = *a* O (by construction), therefore

$$\frac{OP}{OP'} = \frac{OF}{FP'}.$$

Now let u and v denote respectively the distances of object and image from the lens, and f the focal length of the lens, then

$$\mathrm{OP} = u,\quad \mathrm{OP'} = v,\quad \mathrm{OF} = f,\quad \text{and}\quad \mathrm{FP'} = f - v,$$

$$\therefore\ \frac{u}{v} = \frac{f}{f - v},$$

$$\therefore\ uf - uv = vf,$$

$$\therefore\ uf - vf = uv.$$

$$\therefore\ \text{dividing by } uvf,\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \qquad (3.)$$

Case of a convex lens. This case is complicated by the fact that the focal length of a convex lens is a negative quantity.

Two cases arise: (1) When the luminous point is nearer the lens than the principal focus. (2) When further away.

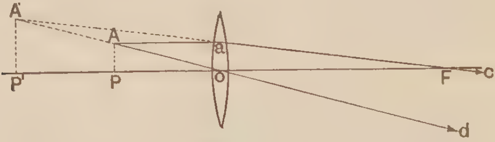

Fig. 92.

Case 1. This is represented in Fig. 92 with letters corresponding to Fig. 91. The image A′ is virtual and further from the lens than the object.

From the similar triangles A P O, A′ P′ O we have

$$\frac{\mathrm{AP}}{\mathrm{A'P'}} = \frac{\mathrm{OP}}{\mathrm{OP'}}.$$

From the similar triangles F a O, F A′ P′ we have

$$\frac{a\mathrm{O}}{\mathrm{A'P'}} = \frac{\mathrm{FO}}{\mathrm{FP'}}.$$

But A P = a O; therefore

$$\frac{\mathrm{OP}}{\mathrm{OP'}} = \frac{\mathrm{FO}}{\mathrm{FP'}}.$$

Now as before let u and v denote the distances of object and image from the lens and f the focal length of the lens. Both u and v are positive in this case, but since f is measured from O in the direction the light is travelling it is negative.

In the fraction $\frac{\text{F O}}{\text{F P}'}$ F O and F P′ stand simply for the numerical values of the lengths of these lines. We must therefore replace F O by $-f$ and F P′ by $v-f$. We therefore get

$$\frac{u}{v} = \frac{-f}{v-f} = \frac{f}{f-v},$$

from which it follows as before that

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \quad \text{.........................} \ (3.)$$

Case 2. This is represented in Fig. 93, with letters corresponding to Figs. 91 and 92. The image is on the other side of the lens and is real.

As above $$\frac{\text{O P}}{\text{O P}'} = \frac{\text{F O}}{\text{F P}'}.$$

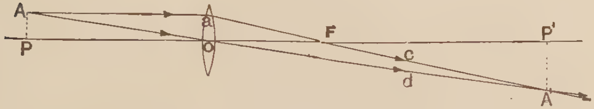

Fig. 93.

Using the letters u, v, f as defined above, we see that in this case u only is positive, v and f being negative. We must therefore replace O P by u, O P′ by $-v$, F O by $-f$ and F P′ by $(f-v)$. (This is because f and v are both negative, but v numerically greater than f.)

We therefore get

$$\frac{u}{-v} = \frac{-f}{f-v}$$

or

$$\frac{u}{v} = \frac{f}{f-v},$$

from which as before we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \quad \text{............................} (3.)$$

This formula, although only proved for points not on the axis of lens, holds for points indefinitely close to the axis, and therefore for points on the axis. It is the most important formula connected with lenses.

As a test of the formula note that when u is infinite, that is when P is at an infinite distance away or the incident light is parallel, we have

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f},$$

but $$\frac{1}{\infty} = 0 \qquad \therefore\ v = f$$

which means that P′ is at the first principal focus.

Now suppose u is altered till v becomes infinity. In this case

$$\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore\ u = -f.$$

In the case of a convex lens (f negative) this means that light diverging from a point in front of the lens and at a distance from it equal numerically to the focal length after refraction from the lens becomes a parallel beam (see Exp. **36**).

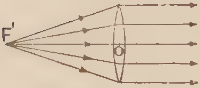

Fig. 94.

Fig. 94 represents the case. F′ is called the **Second Principal Focus.** F and F′ are obviously at the same distance from the lens.

In the case of a concave lens (f positive) it means that light converging upon a point behind the lens and at a distance from it equal to its focal length becomes a parallel beam after refraction from the lens. This is shown in Fig. 95. F′ is again called the **Second Principal Focus.**

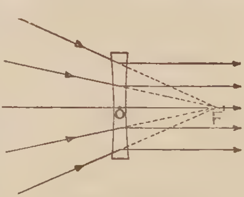

Fig. 95.

If the principal focus is mentioned without any further qualification, the reader should note that usually it is the First Principal Focus that is meant.

Examples. (1) The image of a small bright object placed 20 cms. from a lens is formed at a point 40 cms. *on the other side of the lens.* Find the focal length of the lens and its nature.

Here $u = +20$, $v = -40$, therefore on substituting in the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{-40} - \frac{1}{20} = \frac{1}{f}$$

$$\therefore \frac{1}{f} = -\frac{3}{40} \text{ or } f = -13\frac{1}{3} \text{ cms.}$$

that is, the lens is **convex** and its focal length is $13\frac{1}{3}$ cms.

(2) A small bright object is placed 10 cms. away from a concave lens of focal length 20 cms. Find the position of the image. Is it real or virtual?

Here $u = +10$, $f = +20$; therefore substituting in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get

$$\frac{1}{v} - \frac{1}{10} = \frac{1}{20} \quad \therefore \frac{1}{v} = \frac{2}{20} + \frac{1}{20} = \frac{3}{20} \quad \therefore v = 6\tfrac{2}{3} \text{ cms.}$$

It is on the same side of the lens as the object and is **virtual**.

Change of position in P′ due to change of position of P.

I. **Convex Lenses.**—The following cases admit of simple verification:

(1) When P is at infinity, P′ is at the principal focus (*i.e.* when $u = \infty$, $v = f$) (see Fig. 86).

(2) When P is in front of the lens at a distance numerically equal to twice the focal length, then P′ is at an equal distance behind the lens (*i.e.* when $u = -2f$, $v = 2f$).

(3) When P is at F′, P′ is at infinity (*i.e.* when $u = -f$, $v = \infty$) (see Fig. 94).

(4) When P is at O, P′ is also at O (*i.e.* when $u = o$, $v = o$).

(5) When P is at F, that is when the incident rays converge to the first principal focus, P′ is half-way between O and F $\left(i.e. \text{ when } u = f,\ v = \frac{f}{2}\right)$.

(6) When P is at infinity behind the lens the incident rays are parallel, and, as in (1), P′ is at F (*i.e.* $u = -\infty$, $v = f$) (see Fig. 86).

In cases such as (5) and (6), when the incident rays converge towards a point P and do not actually pass through it, the point of convergence can be described as a virtual object. Similar definitions apply to mirrors, thus in Art. 26A a plane mirror produces a real image of a virtual object.

II. Concave Lenses.

(1) When P is at infinity in front of the lens, P′ is at F (*i.e.* when $u = \infty$, $v = f$) (see Fig. 87).

(2) When P is at F, P′ is midway between F and O $\left(i.e.\text{ when } u = f, v = \frac{f}{2}\right)$.

(3) When P is at O, P′ is also at O (*i.e.* when $u = o$, $v = o$).

(4) When P is at F′, that is when the incident rays are converging to a second principal focus, P′ is at infinity (*i.e.* when $u = -f$, $v = \infty$) (see Fig. 95).

(5) When P is behind the lens at a distance numerically equal to twice the focal length of the lens, P′ is the same distance in front of the lens (*i.e.* when $u = -2f$, $v = 2f$).

(6) When P is at infinity behind the lens the incident rays are parallel and, as in (1), P′ is at F (*i.e.* when $u = \infty$, $v = f$) (see Fig. 87).

Of the above results I., 1, 3, 6 and II., 1, 4, 6 are summed up in the statement that, when a parallel pencil of light is incident on a lens, the focus of the refracted pencil is at the principal focus of the lens. I., 2 is important and should be remembered.

General Rule. When an image is formed by *refraction through a lens*, any motion of the object along the principal axis produces a corresponding motion of the image in the same direction along the axis.

The application of this rule is simple. For example, from I., 1 and 3, we see that, as P moves from infinity to F′, P′ moves in the same direction from F to infinity behind the lens. Similarly from II., 2 and 3, as P moves from F to O, P′ moves in the same direction from a point midway between F and O to O.

Example. Draw a curve showing the relation between the distance of an object and that of its image as measured from a thin lens, as the distance of the object is progressively varied.

Before proceeding to draw the curve draw up a table as shown. In it, d_1, d_2, f_1 are the distances of object and image from the lens, and the focal length respectively. No signs are prefixed in the table, but the position of the object and image whether in front or behind the lens is given.

	d_1	d_2	
In front	∞	f_1	Behind
	$< \infty > 2f_1$	$> f_1 < 2f_1$	
	$2f_1$	$2f_1$	
	$< 2f_1 > f_1$	$> 2f_1 < \infty$	
	f_1	∞	
	$< f_1 > \frac{f_1}{2}$	$< \infty > f_1$	In front
	$\frac{f_1}{2}$	f_1	
	0	0	

The curve is similar to that obtained in the example on pp. 71 and 72.

76. Relation between focal length, radii of curvature of faces of lens, and the refractive index of the glass. It is proved in higher books on this subject* that if r is the radius of the face on which the light first falls, s the radius of the other face, r and s being subject to the sign convention of Art. 39, and μ the refractive index of the glass, f the focal length is given by the formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \text{ .... } (4.)$$

It is thus evident that when we wish to draw accurate diagrams of lenses of known focal lengths it is not only necessary to know the curvatures of the faces but also the refractive index of the glass. But in the case of lenses of *crown glass* there is a simple rule which expresses with a near approach to accuracy the relation of focal length to curvature. The refractive index of crown glass is very nearly equal to 1·5 (Art. 50), therefore on substitution in the general formula—

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* See *Text-book of Light*, 2nd edition, Art. 82.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

we obtain—

$$\frac{1}{f} \approx \frac{1}{2}\left(\frac{1}{r} - \frac{1}{s}\right).^*$$

In the case of a double convex lens whose faces are of the same curvature $s = -r$ and therefore—

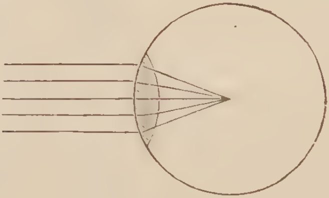

Fig. 96.

$$\frac{1}{f} \approx \frac{1}{2} \cdot \frac{2}{r} \approx \frac{1}{r};$$

that is—

$$f \approx r$$

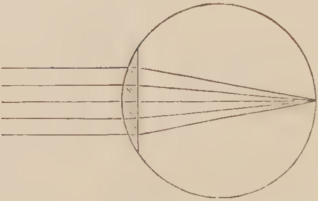

Fig. 97.

or the principal focus of the lens is approximately at the centre of a sphere of which the front surface of the lens forms a part (Fig. 96).

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[*] $\approx$ means "approximately equal to."

In the case of a plano-convex lens $\frac{1}{f} \approx \frac{1}{2} \cdot \frac{1}{r}$ or $f \approx 2r$; that is, if the curved surface is in front, the focus is on the circumference of the sphere of which the front surface is part (Fig. 97). Similar relations exist for double equi-concave and for plano-concave lenses.

77. General construction for images formed by lenses. Let A B (Figs. 98 and 99) represent a small object, say an arrow, placed on the principal axis of a lens O. The position of A′, the image of the point A, is determined, as in Art. 75, with the addition of a ray from A through F′, which after refraction at the surfaces of the lens travels parallel to the principal axis. It is easy to see that the image of any point in A B lies on the perpendicular A′ B′ drawn from A′ to the axis of the lens, and the image of B is therefore at B′. When the rays really intersect at a point, as in Fig. 98, the image formed is said to be real, but when they only apparently intersect at a point, as in Fig. 99, the image is virtual. A real image is always formed on the side of the lens opposite to that on which the object is placed, and

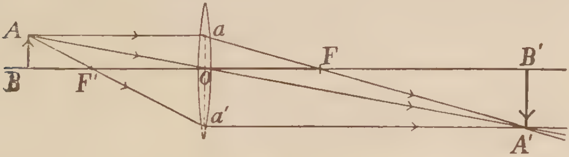

Fig. 98.

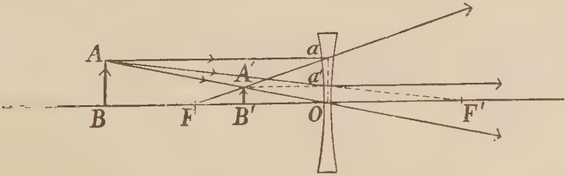

Fig. 99.

may be received on a screen, or seen by an eye* so placed as to receive *the rays involved in its formation.* A virtual image, having no real existence, cannot be said to be *formed* anywhere, but it is always seen on the side from which the light comes by an eye placed on the opposite side of the lens. A virtual image cannot be received on a screen.

78. Relative position of image and object. The formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

deduced for conjugate foci, evidently establishes a relation between the distances of the object and image from the centre of the lens; for an image is an assemblage of foci, conjugate to corresponding points on the object.

The variation of the position of the image with that of the object may be traced by the same method as that adopted in Art. 75 for conjugate foci.

The following cases should be noted:—

I.—Convex lenses.

1. A real object, at a distance from the lens greater than the focal length, has a real image, also at a distance from the lens greater than the focal length, and it is inverted. Fig. 98 illustrates this case; A B represents the

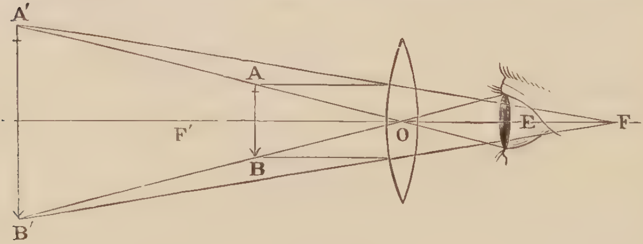

Fig. 100.

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* The eye must be at not less than the least distance of distinct vision from the image. (See Art. 129.)

object, and A′ B′ the image. The magnification is numerically greater than 1 if the distance of object from mirror $<$ twice the focal length.*

2. If the object is real, and its distance be less than the focal length,* the image is on the same side of lens as object, and is virtual, erect, and magnified. Fig. 100 shows this case.

3. If the object is virtual, the image lies between the lens and the object, and its distance from the lens is less than the focal length.* It is real, erect, and diminished.

II.—Concave lenses.

1. A real object always has a virtual erect image, nearer the lens than the object and diminished, and always nearer the lens than its principal focus. (Fig, 99 illustrates this case.)

2. A virtual object, nearer the lens than the focal length, has a real, magnified, erect image, more distant from the lens than the object. (Fig. 99 shows this case if the direction and the rays be reversed.)

3. A virtual object further away than the focus on the negative side of the lens has a virtual inverted image further from the lens than the focus on the positive side.

I., 1, 2, 3 correspond respectively to II., 3, 2, 1. It should be noted that when the object is real, both lenses and mirrors form images which are erect when virtual, and inverted when real.

79. Relative size of image and object. The magnification produced by a lens is expressed by the ratio $\frac{\text{image}}{\text{object}}$. When the image is erect, the ratio is considered to be positive; when inverted, the ratio is taken as negative. In Figs. 98 and 99, let A B represent the object and A′ B′ the image.

1. From the triangles A O B, A′ O B′ we have—

$$\frac{A'B'}{A\,B} = \frac{O\,B'}{O\,B} = \frac{v}{u}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{v}{u}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 5\ (a)$$

both sides of the equation being of negative sign in Fig. 98.

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* In these cases the numerical value of the focal length is taken, *i.e.* the sign of *f* is not considered.

2. From the triangles O F′a′, B F′A we have—

$$\frac{a' \mathrm{O}}{\mathrm{A\,B}} = \frac{\mathrm{O\,F'}}{\mathrm{B\,F'}} = \frac{\mathrm{O\,F}}{\mathrm{O\,B - O\,F'}}$$

but—

$$a' \mathrm{O} = \mathrm{A'\,B'},$$

$$\therefore \frac{\mathrm{A'\,B'}}{\mathrm{A\,B}} = \frac{\mathrm{O\,F'}}{\mathrm{O\,B - O\,F'}} = \frac{-f}{u+f}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{f}{u+f}, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 5\,(b)$$

account being taken of the sign.

3. From the triangles F B′A′, F O a we obtain—

$$\frac{\mathrm{A'\,B'}}{\mathrm{O}\,a} = \frac{\mathrm{F\,B'}}{\mathrm{F\,O}} = \frac{\mathrm{F\,O - B'\,O}}{\mathrm{F\,O}},$$

but—

$$\mathrm{O}\,a = \mathrm{A\,B},$$

$$\therefore \frac{\mathrm{A'B}}{\mathrm{A\,B}} = \frac{\mathrm{F\,O - B'\,O}}{\mathrm{F\,O}} = \frac{f-v}{f};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f}. \quad\ldots\ldots\ldots\ldots\ldots\ldots 5\,(c)$$

Thus we have—

$$m = \frac{\textbf{Image}}{\textbf{Object}} = \frac{v}{u} = \frac{f}{u+f} = -\frac{v-f}{f}. \quad\ldots\ldots\ldots\ (5)$$

These three relations may be very simply connected by the general relation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

by eliminating either u or v. To prove the identity of 5 (b) and 5 (a), v must be eliminated from the general equation. Multiplying both sides of the equation by u we get—

$$\frac{u}{v} - 1 = \frac{u}{f}.$$

$$\therefore \frac{u}{v} = 1 + \frac{u}{f} = \frac{u+f}{f}.$$

$$\therefore \frac{v}{u} = \frac{f}{u+f}.$$

From relation 5 (b) the relative sizes of image and object may be determined without finding the position of the image.

Considering the relation—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}$$

the following results are readily deduced—*

1. If $v > u$, the image > the object.
2. If $v = u$, the image = the object.
3. If $v < u$, the image < the object.

Applying those results to the general case I., 1, of Art. 78, we get three particular cases, according as the image is *magnified*, *equal to the object*, or *diminished*. In the second case, where the image is equal to the object, we have, for a *convex* lens, $v = -u$, and therefore in the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get—

$$-\frac{2}{u} = \frac{1}{f} \text{ or } u = -2f.$$

That is, when the image formed by a convex lens is equal to the object, the distance of both image and object from the lens is equal to twice the focal length of the lens, and therefore the distance between the object and the image is equal to four times the focal length of the lens. It may also readily be proved that this distance is the minimum distance between object and image.*

It may also be noted from I., 1, that in order to produce a magnified image the object must be placed at a distance from the lens numerically greater than the focal length but less than twice the focal length of the lens. *

Examples. (1) An object is placed 12 inches from a convex lens of 8 inches focal length. Find the position and nature of the image.

Here, in formula (4),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

---

* Notice that in these paragraphs, as in many others where no confusion is likely to arise, on matters of sign, the sign convention of Art. 39 is not obeyed.

we have $u = 12$ inches, $f = -8$ inches (*convex* lens), and v is required—

$$\therefore \frac{1}{v} - \frac{1}{12} = \frac{1}{-8},$$

$$\therefore \frac{1}{v} = -\frac{1}{8} + \frac{1}{12} = -\frac{1}{24},$$

$$\therefore v = -24 \text{ inches},$$

that is, the image is 24 inches on the other side of the lens.

Again, applying (5*a*), we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-24}{12} = -2;$$

that is, the image is twice the size of the object, and is *real* and *inverted*.

2. An object, 3 cms. long, is placed 10 cms. from a concave lens of 20 cms. focal length. Find the size and nature of the image. Here from (5*b*) we get—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u+f} = \frac{20}{10+20} = \frac{2}{3}.$$

$$\therefore \frac{\text{Length of image}}{3 \text{ cms.}} = \frac{2}{3}.$$

$\therefore$ Length of image = 2 cms., and the image is *virtual* and *erect*.

A more usual, but less direct method of working this question is, first to determine v, and then to determine the size and nature of the image from formula (5*a*).

3. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed?

[In problems such as this, where by reflection and refraction the image of the object is made to coincide with the object itself, the solution is easy, if we remember that rays diverging from a point in the object, *on the principal axis*, return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflection at a mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror*.]

In this case we know that, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its centre of curvature. To find the position of the object we have therefore only to find a point on the principal axis such, that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror.

Hence in the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have—

$$v = 6 \text{ inches}, f = 12 \text{ inches, and } u \text{ is unknown.}$$

$$\therefore \frac{1}{6} - \frac{1}{u} = \frac{1}{12}.$$

$$u = 12 \text{ inches.}$$

That is, the object must be placed 12 inches from the lens on the side remote from the mirror.

80. Exercise. To construct the rays formed by refraction in a lens by which an image is seen by the eye. If the object be near the principal axis the eye should be near the axis; it will then receive rays which have penetrated the lens. Fig. 101 illustrates the case of a convex lens yielding a real image. A B is the object, A′ B′ the image (found by Art. 77). To find the course of the rays by which A is seen, join A′ to the extremities of the pupil of the eye, then produce the lines so obtained backwards to cut the lens in *a a* and join *a a* to A. The rays by which A is seen are all included in the incident pencil A *a a*. Similarly for B. Fig. 100 illustrates the case of a virtual image formed by a convex lens, and Fig. 102 the case of a virtual image formed by a concave lens.

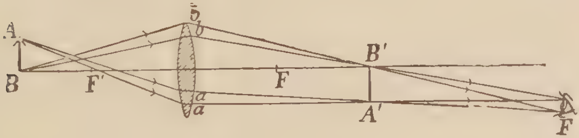

Fig. 101.

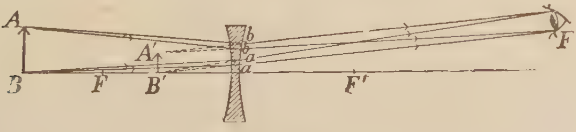

Fig. 102.

Example. When a luminous point is placed on the principal axis of a convex lens (A) and at a distance *a* from it an image is formed 10 inches from the lens on the other side, if a second lens (B) is placed close to A, the image is 15 inches off. Determine the focal length of the lens B, and state whether it is concave or convex.

The action of the lens B is evidently to cause a pencil of rays, originally converging to a point P, 10 inches behind the lenses, to become less convergent, and to converge to a point P′, 15 inches behind the lenses. Thus P and P′ are conjugate foci with respect to the lens B, P′ being the image of P, and, therefore, in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have $u = -10$ inches, $v = -15$ inches, and f is unknown:

$$\therefore \frac{1}{f} = -\frac{1}{15} - \frac{1}{-10} = -\frac{1}{15} + \frac{1}{10} = \frac{1}{30},$$

$$\therefore f = 30;$$

that is, the lens is *concave*, and its focal length is 30 inches.

81. Combination of lenses in contact. Let two thin lenses of focal lengths f_1 and f_2 be placed in contact. The problem is to determine the focal length of a single lens which is optically equivalent to this combination (Fig. 103).

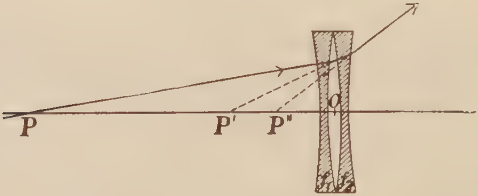

Fig. 103.

Imagine light from a point P, at a distance u from O, the centre of the combination,* to be incident first on the lens of focal length f_1. Then, *considering the action of this lens only*, the focus of the refracted pencil will be at a point, P′, at a distance v' from the lens, such that we have—

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}. \tag{1}$$

---

* The thickness of the lenses is supposed to be so small, compared with the other distances involved, that the centre of the combination may be taken at any point in their combined thickness.

But this refracted pencil passes through the second lens, and after doing so is refracted through another point P″, at a distance v from the lens, such that—

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}. \qquad (2)$$

The combined action of the lenses is thus to cause a pencil diverging from P, at a distance u from the centre, to be refracted through P″, at a distance v from the centre. Therefore, if F be the focal length of the combination, we have—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}. \qquad (3)$$

But by adding (1) and (2) we get—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}. \qquad (4)$$

Therefore, from (3) and (4) we get—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

$$\therefore F = \frac{f_1 f_2}{f_1 + f_2};$$

that is, a single lens of focal length $\frac{f_1 f_2}{f_1 + f_2}$ is optically equivalent to two thin lenses in contact, and of focal lengths f_1 and f_2. When the positions of the equivalent lens and the combination are the same the image produced by the equivalent lens is produced in the same place and is of the same size as that produced by the combination.

By extending the problem to a number of thin lenses in contact we obtain—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots \text{etc.} = \Sigma\frac{1}{f},$$

care being taken in actual work to give the proper signs to the numbers representing the several focal lengths.

82. Power of a lens. The reciprocal of the focal length of a lens expressed in metres is called the **power of a lens.** The unit of power is that possessed by a lens of one metre focal length and is called the dioptrie. The power of a converging lens is considered positive, and that of a diverging lens negative, so that the sign of the power is the reverse of that of the focal length.* The theorem proved in the last article may thus be expressed: **The power of a combination of lenses in contact is equal to the sum of the powers of the constituent lenses.**

Examples. (1) The focal length of a convex lens is 40 cms. What is its power?

To be exact, $f = -40$ cms. $= -\frac{40}{100}$ m.,

$\therefore$ the power $= 1/\frac{40}{100} = 2\frac{1}{2}$.

(2) The focal length of a concave lens is 2 metres. What is its power?

$f = 2$ metres, $\therefore$ the power $= -\frac{1}{2}$.

83. Spherical aberration by refraction. In discussing the formation of foci and images we have only considered cases in which the curvature of the faces of the lens is small. If the faces be greatly curved, rays diverging from any point in the object are not all brought together at one conjugate focus, but those rays which pass furthest from the centre of the lens have a focal distance shorter than that found by the rules we have learnt. This is shown in Fig. 104, where the marginal rays are seen to intersect before the central ones which cross at the principal focus F. This wandering of the intersections of the refracted rays from the focus is called *aberration*. (See also Art. 70.)

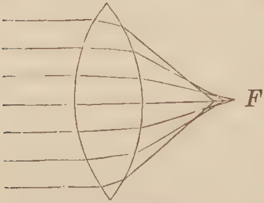

Fig. 104.

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* Sometimes—especially by opticians—a convex lens is taken as positive and a concave lens as negative, in defiance of Art. 72. Students must be on the look out for this possible confusion; the context will sometimes reveal which kind of lens is meant.

Exp. 37. To illustrate the different focal lengths of the central and marginal parts of a convex lens. Take a large convex lens of a short focus and, covering the centre of the lens with a large circle of black paper, use it to throw an image of a candle flame on a screen. Then, without altering the relative positions of candle, lens, and screen, replace the paper disc by a black paper ring which covers all the lens except a small central circle. The image of the flame will now be indistinct, and the screen will have to be moved further from the lens to make the image sharp.

Since spherical curves are always employed, all lenses are subject to this defect in some degree, but if the curvature be small, the defect is very slight. It may be reduced to any required degree by the use of diaphragms (Art. 45), which cover up more or less of the edge of the lens, exactly as in the case of curved mirrors, but obviously what is gained in definition by their use is lost in brilliancy.

The aberration may also be largely diminished by the use of a plano-convex lens, instead of one which is double convex. The curved surface must face the rays which are the more nearly parallel to the axis. Each ray is now very nearly equally deviated at the two surfaces, and higher mathematics proves that under these conditions the aberration is a minimum. This fact is largely utilised in the construction of complicated optical instruments.

The aberration produced by a lens may also be greatly diminished by a suitable choice of the radii of curvature of the surfaces.

84. Caustics by refraction. An inspection of Fig. 104 will show that the intersection of the rays from different parts of the lens must give rise when the curvature is great to a luminous curved surface which is known as a *real caustic by refraction*, just as we found concave mirrors to produce real caustics by reflection.

The presence of a caustic surface affords the easiest method of illustrating spherical aberration to an audience. If the region to the right-hand side of the lens in Fig. 104 is filled with smoke the caustic surface is rendered very apparent.

EXERCISES VI.

1. Show that if the angle of a prism be greater than twice the critical angle for the medium of which it is composed, no ray can pass through it.

2. The angle of a prism is 60°, and the refractive index of its material $\sqrt{2}$. Show that the minimum deviation is 30°.

3. A glass prism of refracting angle 5° is immersed in water; find the approximate deviation produced in a ray of light for which the absolute refractive indices of glass and water are respectively $\frac{3}{2}$ and $\frac{4}{3}$.

4. The minimum deviation produced by a hollow prism, filled with a certain liquid, is 30°; if the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

5. Show that when a ray of light is refracted through a prism, in the position of minimum deviation, the course of the ray in the prism is perpendicular to the line bisecting the angle of the prism.

6. A gas flame is at a distance of 6 feet from a wall. Where must a convex lens, of 1 foot focal length, be placed in order to give a distinct image of the flame on the wall? Explain your result.

7. An object, 1 inch long, is placed at a distance of 1 foot from a convex lens of 10 inches focal length; find the nature and size of the image.

8. If an object, 10 cms. from a convex lens, has its image magnified 4 times, what is the focal length of the lens?

9. An object is at a distance of 3 inches from a convex lens of 10 inches focal length. Find the nature and position of the image.

10. An object is placed 6 inches from a lens, and an image, 3 times as large, is seen on the same side of the lens as the object. Find the focal length of the lens.

11. A convex lens of 6 inches focal length is used to read the graduations of a scale, and is placed so as to magnify them 3 times; show how to find at what distance from the scale it is held, the eye being close up to the lens.

12. The image formed by a convex lens is n times the size of the object. Show that the distance of the object from the lens is $\frac{n+1}{n}f$.

13. A candle flame is placed 6 inches from a plane mirror, and a convex lens, of 3 inches focal length, is placed between the candle and the mirror, and 2 inches from the latter. Find the position of the image.

14. A candle flame is placed 20 cms. from a plane mirror. Find where a convex lens of 5 cms. focal length must be placed in order that the image of the flame may coincide with the flame itself.

15. Show (by application of formula 5a of Art. 79) that the image formed by a concave lens is always smaller than the object.

16. If a candle is placed at a distance of 6 feet from a wall, and a distinct image of the flame is produced upon the wall by a lens held at a distance of 1 foot from the candle, show that a distinct image will also be produced when the lens is at a distance of 5 feet from the candle, and compare the sizes of the two images.

17. The chief focal length of a lens is 12 inches. How far from the lens must a luminous object be placed in order to obtain an image twice as large every way as the object?

18. A circle 1 inch in diameter, a convex lens whose focal length is 6 inches, and a second lens whose focal length is 10 inches, are placed so as to have a common axis. The distance from the circle to the first lens is 20 inches, and from the first lens to the second 36 inches. What images of the circle will be formed, where will they be situated, and what will be their dimensions?

19. An object, whose length is 2 inches, is placed 6 inches in front of a convex lens whose focal length is 4 inches. What is the length of the image?

20. A concave lens, whose focal length is 12 inches, is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens and forms an inverted image coincident with the object itself. Where must the object be placed?

21. An object is fixed 1 foot above the surface of still water. Show how to find the apparent position of this object as seen by an eye 2 feet vertically under it.

22. An object 3 inches in height is placed at a distance of 6 feet from a lens, and a real image is formed at a distance of 3 feet from the lens. The object is then placed 1 foot from the lens. Where and of what height will the image be?

23. A convex lens of 10 inches focal length is combined with a concave lens of 6 inches focal length. Find the focal length of the combination.

24. Find the focal length of a lens which is equivalent to two thin convex lenses of focal lengths 20 cms. and 30 cms. placed in contact.

25. A convex lens of focal length 12 cms. is placed in contact with a concave lens, and the focal length of the combination is found to be -24 cms. Calculate the focal length of the concave lens.

26. A convex lens of power 5 is placed in contact with a concave lens of power -2. Find the power of the combination.

27. What is spherical aberration? How is it produced and how may it be diminished?

28. Plot a curve between the distance of an object from a convex lens and the distance between the object and its real image. Show that the shortest distance between object and image is equal to four times the focal length of the lens, and that object and image are then equally distant from the lens.

CHAPTER VIII.

EXPERIMENTAL DETERMINATION OF THE OPTICAL CONSTANTS OF LENSES, MIRRORS, AND PRISMS.

THIS chapter contains several simple experiments, many of which can be done with very simple apparatus. It is hoped that students will at least attempt some of them, as by so doing the theory of lenses, mirrors, and prisms will be much better understood.

85. Discrimination between long-focus lenses and plates of glass. If the focal length of a lens is short, the curvature of the surfaces is very apparent, and by mere fingering the lens can be classified as convex or concave. But if the focal length is long, the surfaces are very nearly flat, and it may be difficult to distinguish the lens from a plate of glass, and if a lens, to decide whether it is convex or concave.

Exp. 38. To detect a plane surface, hold it in a horizontal position near to and just below the level of the eye, and observe in it the image of the horizontal bars of a window frame reflected at nearly grazing incidence. If the images are straight, the surface is plane; if distorted, the surface is curved. If, further, the ends of the image are bent towards the observer the surface is convex, and if bent away the surface is concave.

Exp. 39. To settle the question of the nature of a lens, hold it up just in front of the eye, and view a distant object through it. Move the lens to and fro across the line of sight; if the image appears to travel across the lens in the same direction as the lens is moving, the lens is concave; if the directions of motion are different, the lens is convex. Figs. 105 (A and B) illustrate the method. The rays coming from the distant object O are sensibly parallel, and after refraction at the convex lens converge to the principal focus F. The focal length is long, and the eye, which may be considered to be at E, is therefore well inside F. E sees the object by means of the ray O A E F. When the lens is shifted to L′, the object is seen by means of the ray O A′ E F′, and from the diagram it is plain that the motion of A to A′ is in an opposite direction to that of L to L′—*i.e.* the object appears to travel across the lens in an opposite direction to that in which the lens is moving. Fig. 105, B, illustrates the case of a concave lens, and shows that the lens and image move in the same direction.

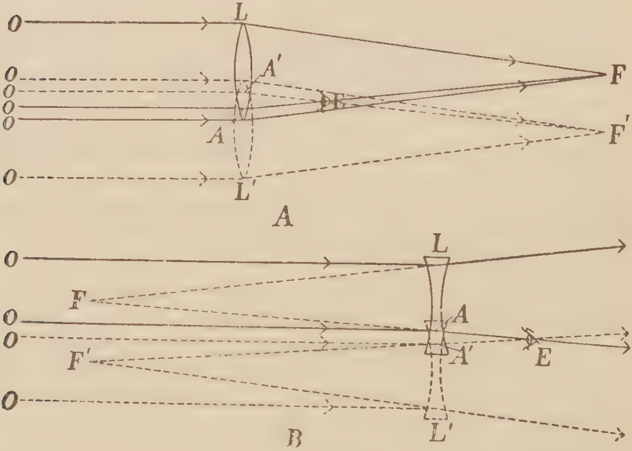

Fig. 105.

This method is only applicable, and indeed it need only be used, for convex lenses when the focal length is large and E can be placed between the lens and its principal focus. If the real image of an object be viewed through a convex lens, it will be found that lens and image move in the same direction.

86. Determination of the focal length of a lens. The experimental determination of focal length is of great importance. The methods adopted depend upon the nature of the lens, and upon the degree of accuracy required. We shall consider a few of the simpler approximate methods for each class of lenses.

I. Convex lenses.

1. The simplest method of determining the focal length of a convex lens is to allow a beam of parallel light to be incident on the lens, in a direction parallel to the principal axis, and then to measure the distance of the focus of the refracted pencil from the lens.

Exp. 40. For this purpose mount the lens in a suitable stand or clip, with its axis parallel to a graduated wooden rod, along which the stand slides. At one end of this bar, and at right angles to its length, fix a white cardboard screen, with its centre approximately on the same level as the principal axis of the lens. Point this arrangement towards the sun or some other well-defined distant object, and adjust the position of the lens by the method of oscillations (Exp. 6) until a clearly defined image of the sun, or other object chosen, is formed on the screen. The distance between the lens and the screen, as indicated by the graduations on the rod, gives the required focal length.

2. This method is based on the fact that if a beam of parallel rays leaves the lens, the source of the rays must be at the principal focus.

Exp. 41. Focus a telescope T (Fig. 106) on a distant object. Fix the lens L to the telescope, in front of, and coaxial with, the object glass. Now place a sheet, P, of printed matter in front of the lens, and move it to and fro along the line of sight until the print is clearly seen by an eye looking through the telescope. Since the telescope is set for parallel rays, P must be at the principal focus of L.

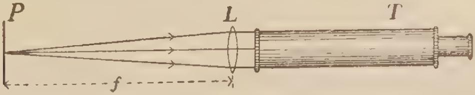

Fig. 106.

3. This method is a combination of methods (1) and (2). A source is placed at the principal focus of a lens. The parallel beam resulting on refraction is reflected by means of a plane mirror, and re-traversing the lens is brought to a focus alongside the object.

Exp. 42. Take a simple form of optical bench, lamp, and plane object as described in Art. **46**; place the lens L (Fig. 107) in a clip, and close behind it place a plane mirror M. Move the lens and mirror about until the rays returned by reflection at the mirror form an image of the gauze on the screen by the side of the gauze itself.* Employ the method of oscillations (Exp. 6) in order to get the best position. Measure O L; it is the focal length, for, since

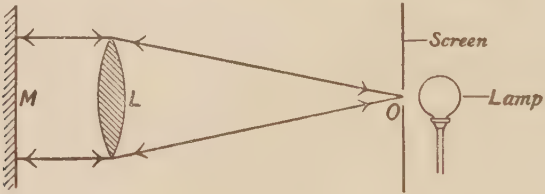

Fig. 107.

the rays return along their old paths, they must have been *directly* incident on M, and have therefore left L as a parallel beam. It will be necessary to slightly tilt M to one side in order that the image may be observed apart from the object. This method is perhaps the simplest method, and is especially useful in the case of long-focus "convex" lenses. This and nearly all the following experiments should be carried out in a darkened room.

4. This method is an application of the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Exp. 43. Mount the lens in one of the uprights of the optical bench, with its axis parallel to the length of the bench. In two other uprights, placed one on each side of the lens, fix a lighted candle and a screen, the flame and the centre of the screen being adjusted to the level of the principal axis of the lens. By properly adjusting the positions of these two uprights, relative to that carrying the lens, focus a sharply defined image † of the flame on the screen. Determine the distances v and u by noting on the scale of the bench the distances between the screen and the lens and between the candle flame and the lens. Substitute these values in the above equation, taking care that v is represented by a negative number. Measurements should be made for several different values of v and u, and the mean of the results taken as the mean value of f.

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* An image may be produced by reflection from the lens surfaces. In the case under consideration the right image disappears if the mirror is taken away.

† The image should be as little coloured as possible. (See Art. **99**.)

A candle will do in a rough experiment, but if accuracy is required use illuminated gauze or cross wires as in Art. **46.**

If, in the general formula above, we replace u by d_1, v by $-d_2$, and f by $-f_1$, where d_1, d_2, f_1 are simply the numerical values of $u, v,$ and f, we obtain the formula

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f_1},$$

which is simpler for purposes of calculation; the arithmetic is still further simplified if tables of reciprocals are employed. The results of the experiment may be plotted on squared paper. Take values of d_1 along O X and values of d_2 along $O\,Y_2$ (Fig. 108). Join up corresponding points. All these lines should intersect one another at the point whose coordinates are equal to the focal length.*

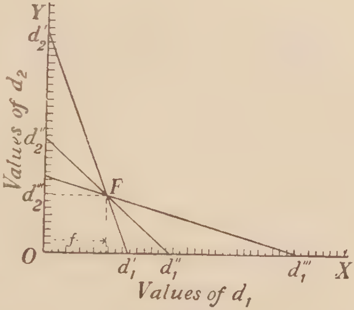

Fig. 108.

5. The displacement method. Let A and B (Fig. 109) represent respectively the positions of a bright object and a screen. Then, if a *magnified* image of the object A be

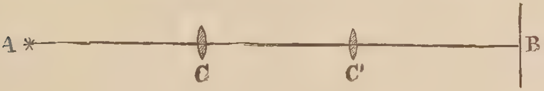

Fig. 109.

formed on the screen at B by a lens placed at C, a *diminished* image can also be obtained on the screen by placing the lens at a point C′ such that B C′ = A C, for, if A C and B C are conjugate focal distances, then the *equal* distances A C′ and B C′ are also conjugate. Let A B be denoted by l

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* See Briggs and Bryan's *The Right Line and Circle*, § 32.

and C C′ by a, then if A C and C′ B be each denoted by d_1, and C B and A C′ be each denoted by d_2, we have—

$$\mathrm{A\,B} = \mathrm{A\,C} + \mathrm{C\,B},$$

$$\therefore \quad l = d_1 + d_2; \qquad (1)$$

and—

$$\mathrm{C\,C'} = \mathrm{A\,C'} - \mathrm{A\,C},$$

$$\therefore \quad a = d_2 - d_1. \qquad (2)$$

Therefore from (1) and (2) we get—

$$d_1 = \frac{l - a}{2}, \text{ and } d_2 = \frac{l + a}{2}$$

and therefore by the formula of Page 165,

$$\frac{2}{l - a} + \frac{2}{l + a} = \frac{1}{f_1}$$

$$\therefore f_1 = \frac{l^2 - a^2}{4l}. \qquad (3)$$

Hence, by measuring l and a, f may, by application of this formula, be readily determined. This method does not involve any error due to inexact knowledge of the position of the centre of the lens.

If a, in formula (3) above, becomes zero, we have, neglecting sign—

$$f = \frac{l^2}{4l} = \frac{l}{4}.$$

When this is the case the points C and C′ in Fig. 109 are evidently coincident, and we have A C = C B. That is, image and object are equidistant from the lens, and are therefore equal in size (Art. 79).

II. Concave lenses.

We have already seen that with a concave lens the image of a real object is virtual, and so cannot be received upon a screen. This renders it difficult to determine the focal length of a concave lens, but the following methods may be adopted with fairly accurate results :—

1. One face of the concave lens is covered with a circular piece of black paper, through which two large pinholes have been made, on a diameter of the circle, at points equidistant from the centre. A beam of parallel light is then

directed on the lens in a direction parallel to the principal axis. All the incident rays except those passing through the pinholes are stopped by the black paper, and if a screen be placed behind the lens, two bright spots will be formed upon it at the point where the rays passing through the pinholes meet its surface.

Let *a* and *b* (Fig. 110) represent the position of the pinholes, then, the incident light being parallel, the rays refracted through at *a* and *b* diverge from the focus F, and bright spots of light are formed, at *a′* and *b′*, on a screen placed at any point S, behind the lens.

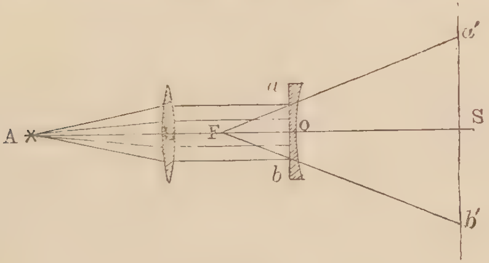

Fig. 110.

From the figure we have—

$$\frac{a\,b}{a'b'} = \frac{\mathrm{F\,O}}{\mathrm{F\,S}} = \frac{\mathrm{F\,O}}{\mathrm{FO+OS}}.$$

Therefore, if the focal length of the lens be denoted by *f*, we get—

$$\frac{a\,b}{a'b'} = \frac{f}{f + \mathrm{O\,S}}.$$

From this relation *f* can be determined when *a b*, *a′ b′*, and O S are known.

Exp. 44. Place the source of light, A, at the focus of the convex lens, and then place the concave lens and screen in position. Measure *a b* and *a′ b′* with a pair of compasses and a fine scale and read off the distance O S from the bench. Only very rough results can be obtained by this method.

2. Let P (Fig. 111) denote the position of the object, and P′ the position of the image formed on the screen at S by the convex lens L. If the *concave* lens be placed at L′ in such a position that L′ P′ is less than its focal length, then the rays converging to P′ become less convergent and thus meet at a more distant point P″ (Art. 78, II., 2). If the screen be placed at S′ an image of the object at P is formed on it; this image may be considered as the image of a virtual object P′, and if L′ P′ and L′ P″ be measured the focal length of the concave lens may be calculated from the relation—

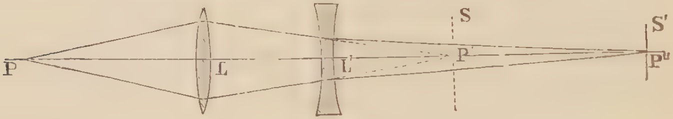

Fig. 111.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Exp. 45. Mount the lens L on the optical bench and locate P′ by means of a screen S. Note its position. Mount L′ on a stand so as to be the same level as L and place it in position. The image is now thrown back to P″. Locate it by the screen as before. If L′ P″ is very long, there will be a considerable range for which the image is approximately in focus. To overcome this difficulty place L′ nearer P′; P″ is now considerably nearer P′.

3. If L′ (Fig. 111) is so placed that L′ P′ is equal to its focal length, the rays leaving L′ will be parallel to one another, and may be made to retrace their paths by means of a plane mirror placed at right angles to them.

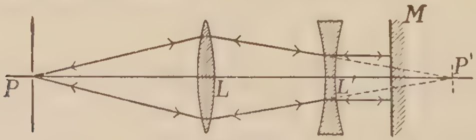

Fig. 112.

Exp. 46. Place a plane mirror M (Fig. 112) behind L′ and move the lens along the bench until an image of the gauze at P is formed alongside P. Note the position of L′ and then remove it and M. The rays now come to a focus at P′; P′ L′ is the focal length of the concave lens.

4. It has been shown in Art. 81 that if two thin lenses of focal lengths f_1 and f_2 be placed in contact so as to act as one compound lens, then the focal length, F, of the combination is given by the relation—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

If a concave lens, of focal length f_1, be combined in this way with a convex lens of *shorter* focal length, f_2, the combination is evidently equivalent to a convex lens, and its focal length, F, may be determined by either of the methods given above. Similarly f_2 may be determined, and the required focal length f_1 may then be calculated from the relation just given. Care must be taken over the signs.

87. Radius of curvature of a mirror or surface of a lens.

I. Concave surfaces.

The concave surface of a lens may be regarded as a concave mirror, and its radius determined as in Art. 47.

II. Convex surfaces.

Art. 47 gives one method; the following is also in general use:—

If a convex surface is so placed in a converging beam that the focus of the beam is the centre of curvature of the surface, the rays are reflected back upon themselves and retrace their former paths.

Exp. 47. Place the surface to be measured (M, Fig. 113) behind a convex lens, L, of short focal length, and adjust their

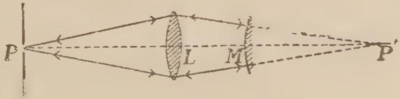

Fig. 113.

positions until a sharp image of the gauze (Art. 46) is formed on the screen by rays returned after reflection at the convex surface.

Note the position of M, then remove it; the rays now travel on and an image of P is formed at P′. Since the rays were incident directly on M they were converging to its centre of curvature, hence M P′ is the radius of curvature. This method may also be used to find the radius of the convex surface of a lens; the second surface will not interfere.

88. Determination of the refractive index of the glass composing a lens. The focal length and radii of the two surfaces being found by the experiments described above, μ can be found from the formula—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

in which r is the radius of the surface upon which the light first falls. Great care must be taken over the signs.

89. Experimental illustration of the deviation produced by a prism. In Arts. 67, 68, the phenomenon of deviation of light by a prism was illustrated by the displacement of the *virtual* image of a slit seen through the prism. We can, however, illustrate deviation by noticing the displacement of a *real* image, obtained by means of a lens and prism. The apparatus described below may be employed for this purpose, and will also serve to give rough measurements of the deviation produced.

Exp. 48. A small truly circular table, about 30 cms. in diameter, is fitted round its edge with a strip of thin tough paper, in such a way that the upper edge of the strip projects about 5 cms. above the face of the table. On this projecting edge, about half-way up, a scale, showing degrees, is marked all round the strip. At the 180th division on the scale, a narrow vertical slit, S (Fig. 114), about 2 cms. long and ·5 mm. wide, is so cut in the paper that one edge accurately coincides with this division. Take the apparatus into a dark room and illuminate the slit by a properly shaded sodium flame.* Fix a mounted convex lens, L (f_1 = 5 cms., about), on the table, between the slit and the centre, in such a position that a clearly defined image of the slit, having one edge coincident with the zero on the scale, is obtained on the paper strip at S′.* At the centre of the table fix a small stand T T, which can be rotated round a central vertical axis. On this stand place a prism so that its edge is vertical, and the plane bisecting its refracting angle passes through the axis of rotation of the stand. Rotate the stand until the rays of light coming from the slit, through the lens, are refracted through the prism ; observe that the position of the image of the slit is changed, and that the change of position indicates that the rays are deviated by the prism in a direction away from its refracting edge. Note also that as the rotation continues the position of the image changes, indicating that the magnitude of the deviation produced depends upon the position of the prism relative to the incident light. If for any position of the prism the image is formed at S″, then the magnitude of the deviation is measured by the angle S′ P S″, which may be at once read off on the scale. Now rotate the prism so as to cause the deviation to diminish. As the prism is rotated, always in the same direction, the image will travel at a gradually decreasing rate towards S′, and at a certain point will become stationary, and then turn back in the opposite direction. The deviation at the

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* To set up a sodium flame, take about a foot of iron wire and wind one end in a conical spiral. Wind the other end round the tube of a Bunsen burner and place a piece of salt on the upper spiral so that the flame comes in contact with it.

Another way is to soak a piece of asbestos paper in strong brine, wrap it around the top of the burner, and fasten it in position with wire.

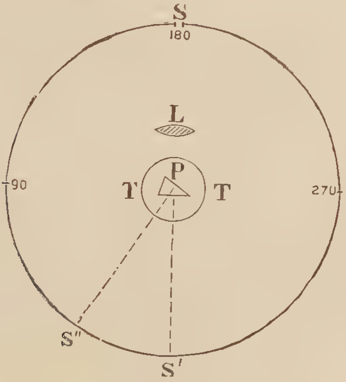

Fig. 114.

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* If the lens be not employed the light leaves P in diverging pencils and an indefinite image is formed. With the lens in the correct position the pencils converge as in Figs 115, 119 and form a real image on the circular scale

instant at which the image is stationary is the *minimum deviation*, which can thus be determined by noting, on the scale, the division at which the image ceases to advance towards S′ and begins to turn back. The image obtained on the scale, after the interposition of the prism, is not clearly defined except at, and near, the position of minimum deviation, and consequently measurements made near this position will be more correct than for other positions. Accurate measurements of deviation are made by means of the *spectrometer*. (Art. **144**.)

90. Exp. 49. Determination of the angle of a prism. (1) Take the simple apparatus of the last Article, and having adjusted L in position, place P on the stand so that the refracting edge is over the axis of the rotation of the stand, and the faces O R, O T are nearly symmetrical about S O (Fig. 115). The converging beam of light falling on P is now reflected in two portions, and the reflected beams converge to points A and B on the scale. Read off on the scale the magnitude of the angle A O B. It is twice that of the angle of the prism, for by Art. **30** the

$$\angle AOS' = 2\angle ROS'$$
$$\angle BOS' = 2\angle TOS';$$

hence
$$\angle AOB = 2\angle ROT.$$

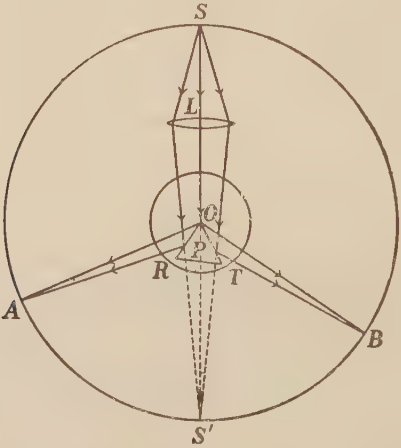

Fig. 115.

(2) *A pin method* is still simpler. Place the prism O R T on a piece of paper and stick a pin S into the paper in a position as in

Fig. 116, the pin being *as far away as possible*. With the eye near A look at the reflection of S in the face O R, and move the eye until this image apparently gets very near to O. Stick pins A and P into the paper to be in line with this image. Similarly stick pins Q and B into the paper to be in line with the image of S in the face O T when this image is apparently as near as possible to O. Now remove the prism and pins. Join A P and B Q and produce them to intersect. The point of intersection is very near O. Call it O. Then the ∠ A O B is approximately equal to twice the angle R O T. The proof is similar to the above. The ∠ A O B is in this case less than twice the angle R O T by an amount equal to the angle at S, but if S is far away this angle is very small.

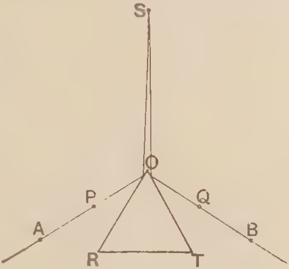

Fig. 116.

Check the result by marking the outline of the prism on paper and measuring the angle R O T with a protractor.

91. Determination of the refractive index of the substance of a prism. Having now determined the refracting angle of the prism and the minimum angle of deviation, we have all the data necessary for the calculation of the refractive index of the material of the prism for sodium light. For if A is the refracting angle, and D the minimum angle of deviation,

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}} \qquad \text{(Art. 66).}$$

The refractive index of liquids can be measured by enclosing them in hollow prisms whose walls are made of thin parallel plates of glass suitably cemented together at their edges.

Rectangular pieces of thin plane glass should be cut of different length but of same height, and their vertical edges bevelled off at suitable angles. The base plate and cover should then be cut; these should be slightly larger than the triangle formed by the vertical sides. The cement used must, of course, be insoluble in the liquid to be used in the prisms; for a water prism bicycle cement is convenient, while for a carbon bisulphide prism glycerine glue may be used.

EXERCISES VII.

1. In an experiment made to determine the focal length of a convex lens by the method of Art. 86, I. 4, the following corresponding values of d_1, d_2 were observed:

d_1	cms. 52·5	cms. 38·4	cms. 32·5	cms. 28
d_2	30·5	41·4	51·9	69·5

Find by a graphical method the focal length of the lens.

2. An object and screen were fixed on an optical bench at a distance apart of 94·1 cms. Between them a convex lens was moved about, and in two positions, 71·3 cms. apart, images were formed on the screen. Find the focal length of the lens.

3. A contact-combination of a convex and a concave lens had a focal length of − 19·3 cms. The focal length of the convex lens being − 10·02 cms., find that of the concave lens.

4. A convex lens was used to form a real image of an object. Between this image and the lens, and at a distance of 29·25 cms. in front of the image, a concave lens was placed, and it was found that the image was thrown back 12·25 cms. Find the focal length of the concave lens.

5. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 150 cms. from itself. At 50 cms. behind the lens was placed a concave lens, and behind this a plane mirror, and it was found that an image of the gauze was thrown back on the gauze itself. Find the focal length of the concave lens.

6. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 40·38 cms. from itself. An equi-convex lens was placed behind this lens, and it was found that an image of the gauze was found alongside the gauze itself when the second lens was 29·68 cms. behind the first. Draw the diagram, and find the radii of curvature of the surfaces of the second lens.

Given that the focal length of this lens is − 10·1 cms., find the refractive index of the glass of which it is composed.

7. A hollow glass prism of refracting angle 39° 33′ was filled with water and set on the prism table of a simple spectroscope (Art. 89). The angle of minimum deviation for sodium light was found to be 13° 57′. Find the refractive index of water.

8. A similar experiment was performed with carbon bisulphide. Find its refractive index from the following data: Refracting angle of prism, 40° 24′; angle of minimum deviation, 28°.

EXAMINATION QUESTIONS.

1. A ray of light incident upon a glass prism makes an angle of 60° with the normal to one face, and emerges parallel to the normal to the other face. If the angle between the faces of the prism is 30°, find the refractive index of the glass.

Would it be possible to find an angle of incidence such that the refracted ray would leave the second face normally in a prism of any angle?

2. What is the position of *minimum deviation* for a prism? Describe and explain the appearance presented when the image of a window is looked at through a prism with its edge vertical.

3. When does a prism produce minimum deviation in a ray crossing it? Prove the formula $\mu \sin \frac{1}{2} \mathrm{A} = \sin \frac{1}{2} (\mathrm{D} + \mathrm{A})$.

4. Prove that the deviation produced by a thin prism on light which traverses it nearly symmetrically is $(\mu - 1) A$; where A is the small angle of the prism and μ is the refractive index of the material of which it is made. Explain clearly where you assume that the angle A is small. By what experimental disposition would you verify the formula?

5. Explain what is meant by *conjugate foci* in the case of a prism.

6. A thin hollow prism having glass sides and containing only air is sunk in water. Explain its effect on the apparent positions of objects seen through it, and, neglecting the effect of the glass sides, draw the path of a ray incident normally on one face of the prism.

7. Show that, if the angle of a glass prism be made too great, it is impossible for a ray of light to cross the prism.

8. Explain how to draw a figure to represent the formation of a real magnified image of a small object by a lens. If a real image five times as high as the object is to be thrown on a screen at a distance of 36 inches from the object, show what must be the focal length of the lens employed.

9. An object is moved from a considerable distance on the principal axis of a convex lens up to the lens. Find the corresponding changes in the position and size of the image.

10. Describe the two main categories under which lenses may be classified. A lens of water is enclosed in a rectangular envelope of glass. What kind of lens will the combination form?

11. Given the focal length of a lens, show how, by a geometrical construction, to find the position and magnitude of the image of an object whose distance from the lens is given.

12. Distinguish between a real image and a virtual image. Explain the action of a convex lens when used as a magnifying glass. Is the image seen by the eye real or virtual?

13. Explain with the aid of a diagram how an image of the Sun is formed by a convex lens.

Show how to determine by a geometrical construction the diameter of the Sun's image formed by a convex lens of 6 feet focal length, assuming that the Sun's diameter subtends, as seen from the Earth, an angle of ½°. How does the brightness of the image depend upon the size of the lens and its focal length?

14. An empty vase is placed at a distance of 2 feet from a convex lens of focal length 1 foot. Where must a flower be placed, and how must it be placed, upon the other side of the lens so that to an eye suitably placed it may appear to be standing naturally in the vase? Where must the eye be placed?

15. Two convex lenses, each of focal length five inches, are placed at the ends of a straight metal tube ten inches long. The tube is then directed towards a star. Trace upon a diagram the rays passing through the apparatus.

16. A lantern slide is 3¼ inches square and an enlarged image is to be formed by the aid of a lens of 6 inches focal length upon a screen 20 feet distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

17. A double convex lens, whose focal length is 12 inches, throws on a screen an image twice the linear dimensions of the object. What is the distance of the lens from the screen?

18. An eye looks through a convex lens of 3 inches focal length at an object 2 ins. in front of lens. Where is the image?

19. Where must an object be placed so that a virtual image of it may be formed by a convex lens of 1 inch focal length, at a distance of 12 inches from the lens? Where should the eye be placed so as to see this image to the best advantage?

20. State the laws of refraction of light, and show how to deduce from them the position of the (internal) focus that is conjugate to one outside a single spherical surface of a transparent material whose refractive index is μ.

21. Explain how you would determine the focal length of a convex lens by an experiment that could be carried out on an ordinary laboratory table.

22. Define the focal length of a concave, or diverging lens. A convex lens is placed at a distance equal to its focal length immediately in front of a small hole cut in a screen. The hole is furnished with cross wires and brightly illuminated. In front of the lens and in line with it and the cross wires a concave lens is placed. Sketch the path of the rays from the point of intersection of the cross wires through the two lenses.

In what position must a concave mirror (of known radius of curvature) be placed in front of the concave lens in order that the light falling upon it may be reflected back through the two lenses so as to form a sharp image of the cross wires close to them on the screen? How can you determine the focal length of the concave lens from the result?

23. Describe exactly how you would find the radius of curvature of a concave spherical mirror and how you would use the mirror to form (1) a real and (2) a virtual image of a rod. Give a diagram to show how the image is formed in each case.

24. Describe fully how you would measure the focal length of (1) a concave mirror, (2) a convex lens. How would the result depend in each case upon the colour of the light employed?

25. A converging lens having the same focal length as a concave mirror is placed with its axis coinciding with that of the mirror and with its centre at the centre of curvature of the mirror. Rays of light diverge from the principal focus of the lens remote from the mirror and fall on the lens. Draw a diagram to indicate their path through the lens to the mirror, and thence, after reflexion, back through the lens.

What difference would there be if the lens were placed with its centre at the principal focus of the mirror?

CHAPTER IX.

DISPERSION.

Exp. 50. Take a prism, and look through it at any bright object such as the view of a sky through a window, a fire, a sheet of white paper. Observe the brilliant coloration. Enumerate the colours seen and compare them both in number and arrangement with those of a rainbow and a film of oil upon water.

92. Newton's Experiment. The Spectrum. The light coming from the sun was first shown by Newton (1676) to be of a composite character. The experiment by which this fact was demonstrated is known as Newton's experiment, and is worthy of special notice, both on account of the historical interest attached to it, and because of the great importance of the fact which it illustrates.

In its simplest form Newton's Experiment may be performed in the following way. A beam of sunlight is admitted into a dark room through a small circular aperture, A, in a shutter or blind. The beam will be seen in the room as a small pencil of light diverging from A (Fig. 117), and, if allowed to fall on a vertical screen at S, it forms a small elliptical bright spot, which is a rough image of the sun. If now a prism, P, with its edge horizontal be placed, edge downwards, in the path of the beam, the latter will be deviated from its original course, and deflected upwards so as to form an image at S′S″. This image differs from that first formed at S in several important particulars: the vertical diameter is much longer, and, instead of appearing as a bright patch on the screen, it is made up of several coloured bands, arranged horizontally. In fact, the image is made up of several overlapping images, similar in shape to that originally seen at S, but each of a different colour. This shows that the beam of *white light* incident on the prism is, on refraction through the prism, separated into its different coloured constituents, each of which forms its own image on the screen, and thus the many-coloured compound image at S′ S″ is formed. Such compound images are called *spectra.* When a spectrum is formed by decomposition of the solar light, as in the case we have just considered, it is called the solar spectrum, and on a first analysis may be taken as made up of the six colours—red, orange, yellow, green, blue, and violet.* Of these the red rays are the least deviated, and therefore appear at S′, the bottom of the image S′ S″. The violet rays are the most deviated, and are therefore found at S″, the top of the same image. The intermediate rays are arranged in the order given above, from below upwards, between S′ and S″.

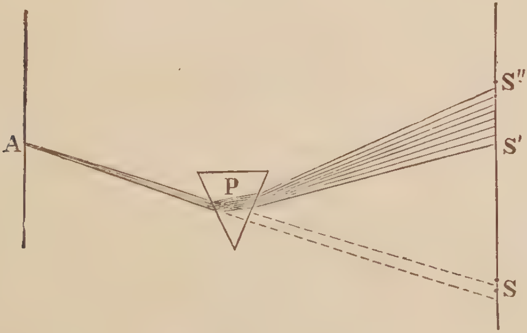

Fig. 117.

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* Newton described *seven* principal colours, including indigo between blue and violet, and others have followed him; but no such colour is to be seen by normal eyes in a pure spectrum, and if it were, indigo is only a kind of blue, and there is no more reason to subdivide the blue than the green or any other colour.

The student must beware of thinking that the spectrum can be divided into six distinct blocks of different colours, and that solar light is made up of only six different constituents, corresponding to the six colours. This is not the case; the number of constituents making up solar light is *infinite*, but, considered with reference to their action on the eye, they may be divided into six sets, each of which corresponds to a definite *colour sensation*, and comprises an infinite number of rays, each corresponding to a certain *shade* of the colour which characterises the set to which it belongs.

93. Homogeneous and compound light. We have seen that there is reason to believe that the physical cause of light is a species of transverse vibratory motion in the ether. When this motion is made up of a series of waves, all of the same wave-length, then the light is said to be *homogeneous* or *monochromatic*. It is, however, more generally the case that the wave-motion is made up of an infinite number of waves of different wave-length. The light is then said to be *non-homogeneous* or *compound*.

Monochromatic light is of a definite colour, corresponding to its wave-length, and difference in wave-length is always indicated by a difference in colour. Compound light may also be of all shades of colour, or may be *white* or colourless, but its colour is no indication of its composition; two lights of almost identical colour may be made up of very different constituents, and may even exactly match the colour of any of the monochromatic lights. Thus the colour of light is not a reliable indication of its composition. In the case of white light, however, we can always state that it is compound, for all monochromatic lights are coloured; but, without experiment, we cannot state what may be the constituents of any given source of white light. Solar light and the other white lights with which we are most familiar—for example, gaslight, lamplight, electric light—are very similar in composition, and include all possible shades of monochromatic light. The reason of this is evident when we remember that white light of this nature always results from incandescence, and that an incandescent or white-hot body has passed through all the phases of change of colour attendant on rise of temperature. It is, therefore, giving out light of all wave-lengths, from the dark red which first appeared when it began to get red-hot, to the violet which was added when it first appeared to be perfectly white.

94. Refrangibility. When the rays of a compound beam of light are refracted through a prism, as in Newton's Experiment, each constituent suffers refraction to a different extent. This is sometimes expressed by saying that the constituents of the compound beam are of different *refrangibilities*; the most *refrangible* rays are those which undergo the greatest deviation, while the least *refrangible* are those which suffer least deviation. In the *solar spectrum* the red rays are the least refrangible, while the violet rays are the most refrangible. The intermediate rays increase in refrangibility as we pass from red through orange, yellow, green, and blue to violet.

From what has been said above, it will be seen that difference in refrangibility corresponds to difference in wave-length. Light of high refrangibility is of short wave-length, and the corresponding index of refraction for any given medium is relatively high, while light of low refrangibility is of long wave-length, and the corresponding index of refraction for any given medium is relatively low.

This difference in refrangibility is due to the difference in frequency of the various coloured rays. Colour, in fact, in light corresponds with pitch in sound. The violet waves are the shortest, the red the longest. A violet light, therefore, corresponds to a high note, a red light to a low note.

95. Pure spectrum. The spectrum obtained by the method of Newton's Experiment is indistinct and badly defined because of the overlapping of the images of which it is composed. Such a spectrum is said to be *impure*. To obtain a *pure* spectrum a very narrow slit must take the place of the aperture in the shutter, and some means must be adopted to obtain a spectrum made up of a series of adjacent but not overlapping images of this slit. In Fig. 118 let s denote the position of the slit. The pencil of rays, diverging from s, forms a broad band $a\ b$ on the screen S S′. If now the prism P be interposed *in the position of minimum deviation*, with its edge parallel to the length of the slit, the rays of the pencil are deviated and dispersed in such a way that the red light appears to come from r', a virtual image of the slit s, and forms a band $r\,r$ on the screen.* Similarly the violet light seems to come from v' and gives the violet band $v\,v$; and so on, for each colour of the spectrum. It is evident that the spectrum thus contained on the screen is composed of a series of overlapping bands, and is therefore not *pure*.

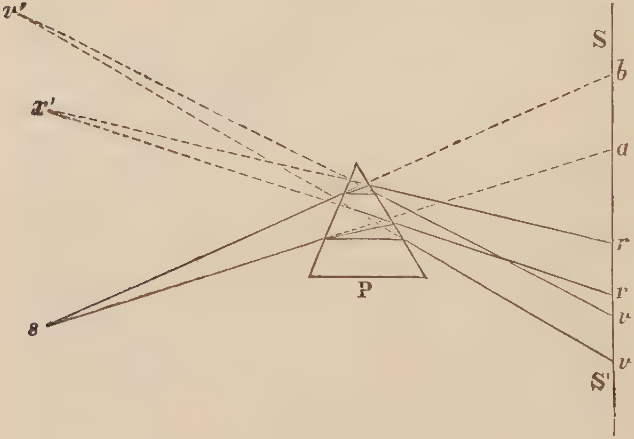

Fig. 118.

If, however, a suitable lens, L, be placed, as shown in Fig. 119, so as to give, when P is removed, a distinct image of the slit at a, and the prism, P, be then interposed, in the position of minimum deviation for the mean rays, the pencil of rays converging to *a* will, after refraction through the prism, be dispersed and give rise to a series of pencils converging to the points *r*, *o*, *y*, *g*, *b*, *v*. Real images of the slit are thus formed at these points of light of each colour, and, as each image is narrow and distinct, like that at *a*, there is no overlapping and a *pure* spectrum is obtained If the slit itself is not sufficiently narrow, the images may be broad enough to overlap and thus give an impure spectrum. Instead of placing the lens at L (Fig. 119), it may be placed on the other side of the prism in such a position (Fig. 120) that real images of the virtual foci lying between *r*′ and *v*′ (Fig. 118) are formed on the screen.

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* In general, oblique refraction does not produce a point image of a point source (Arts. 57 and 67), but in the special case when the prism is symmetrically placed with respect to the incident and emergent rays, the emergent pencil does diverge from a point, and hence we can consider r' and v' as definite images.

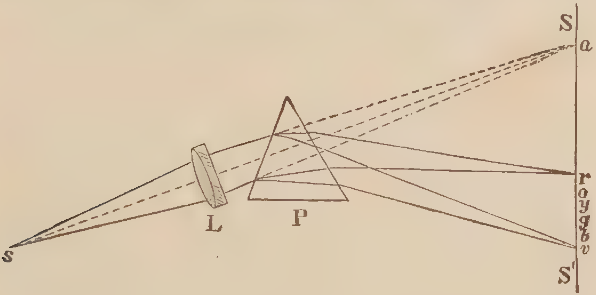

Fig. 119.

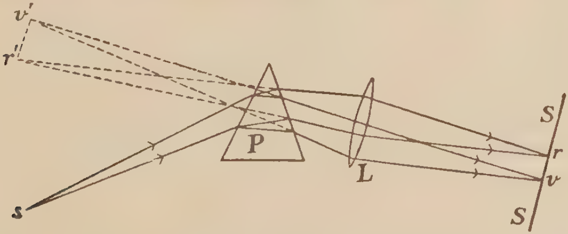

Fig. 120.

A third method of obtaining a real, pure spectrum is described in Chap. XIII. under the spectroscope (Art. 144).

It is thus evident that to obtain a real, pure spectrum we must have—

1. A very narrow slit.
2. The prism in the position of minimum deviation for the mean rays and therefore approximately for all rays.
3. A lens, so placed as to form a *clearly defined* spectrum on the screen.

The second condition is of great importance, for it is only when the prism is in the position of minimum deviation that clearly defined images can be obtained. In practice it will be found convenient to illuminate the slit by means of a lamp, and instead of the large screen shown in the figure, to employ a smaller one, placed first at *a* to receive the direct image of *s*, and then at *r* . . . *v* to receive the spectrum. The prism is placed in the position of minimum deviation by rotating it until the position of the spectrum is as near as possible to *a* (Art. 89).

It should here be noticed that the above arrangement of apparatus is necessary to obtain a *real* pure spectrum, which can be received upon a screen. A *virtual*, pure spectrum can be seen by merely looking through a prism at a narrow slit. An eye placed near the prism, so as to receive the emergent pencil, sees a small but very bright and pure spectrum at the virtual foci of the different constituents of the pencil entering the eye. The violet end of this spectrum is seen nearest the refracting edge, because the violet rays are most refracted. This is evident from Fig. 120, the lens and screen there shown being replaced by the lens of the eye and the retina. The red end of the spectrum is seen at r' and the violet end at v'.

Exp. 51. 1. Cut a narrow slit (about one or two millimetres in width) in a piece of cardboard or metal foil, and place it vertically in front of a bright white light, such as the sun, a bright cloud, an electric arc, a good lamp flame, or an incandescent mantle. Take a glass prism (a glass lustre from a chandelier will do) and stand it in a vertical position in the path of the light issuing through the slit. At some distance behind place a white screen. Note the spectrum formed, its deviation, and the fact that the violet end is deviated more than the red end, and that the centre is very nearly white. Set the prism in the position of minimum deviation.

2. Close behind the prism stand another similar prism also in the position of minimum deviation. Note that the deviation of the spectrum is increased and that it is further drawn out.

3. Take this second prism and now place it horizontally with its refracting edge downwards, close behind the first prism. Note now that the spectrum is elevated, and the violet end more so than the red end so that the spectrum is now on the slant. This is Newton's celebrated *crossed-prisms* experiment.

4. Repeat (1), but instead of using the screen place the eye to receive the emergent beam ; the spectrum seen is nearly pure.

5. Take now a lens of about a foot in focal length and set up the prism and lens as in Fig. 119 or 120. A pure real spectrum is now formed on the screen. Note the purity of the colours compared with those produced in (1).

96. Dispersion. We have seen that when a beam of compound light is refracted through a prism, each constituent of the beam suffers deviation to a different degree. The light of shortest wave-length is deviated most, and that of longest wave-length least, and thus the different constituents of the incident beams are, as it were, separated, each travelling in a definite direction determined by the deviation it has experienced. This separation of the different constituents of a compound beam of light by refraction through a prism is called *dispersion*, and is measured, for any two rays of the refracted pencil, by the angle between these rays. Thus, in Fig. 121, if A B represent a ray incident on a prism at B, and split up by refraction through the prism into a pencil of rays bounded by B C D and B E F, then the angle D O F measures the dispersion for the extreme rays of the emergent pencil.

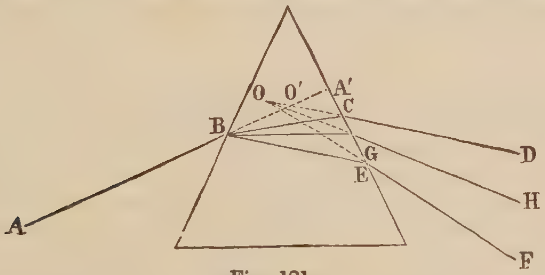

Fig. 121.

97. Dispersive power. The dispersive power of a medium is determined approximately by the ratio of the *extreme* dispersion produced by a prism of small refracting angle of that medium to the *mean* deviation produced by the same prism, when a beam of white light is refracted through it, in the position of minimum deviation.

Thus, in Fig. 121, the dispersion is measured by the angle D O F and the *mean* deviation by A′ O′ H; therefore, in accordance with our definition,* we have—

$$\text{Dispersive power} = \frac{\text{D O F}}{\text{A}'\text{O}'\text{H}} = \frac{d}{\text{D}}, \text{ say.}$$

The dispersive power is usually denoted by ω.

Remember that the dispersive power relates to the material of the prism and not to the prism itself, but for prisms of the same material $\frac{d}{\text{D}}$ is a constant as long as the angle of the prism is small ($< 10°$).

Experiment shows that in general the dispersive powers of different media are different. Water has little dispersive power, crown glass has about the same, flint glass has nearly twice as much as crown glass, and carbon bisulphide has still more, and therefore hollow prisms filled with this liquid are usually employed in spectrum work for lecture illustration.

98. Achromatic prisms. To realise the full significance of dispersion let us consider the following experiments :—

(1) Let two exactly similar prisms P and P′ (Fig. 122), *of the same material*, be placed as shown in the figure, with their refracting edges turned in opposite directions and their adjacent faces parallel, and let a beam of solar light A B be incident on P. On refraction through P, this beam is dispersed, and the refracted pencil, C D, after emergence from P, is incident on P′. Now P′, being exactly similar to P, but having its edge turned in opposite direction, will produce an equal and opposite effect to that produced by

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* In Fig. 121 the angle of the prism is intentionally made large for the sake of clearness.

P; that is, the pencil, E F, after emergence from P′ will be parallel to A B, and the dispersion produced by P will be destroyed by P′, so that the beam E F will be, in all respects, exactly similar to A B. In fact, the action of the combined prisms is the same as a *plate* of the same medium. (Art. 50.)

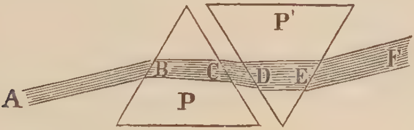

Fig. 122.

This experiment, which may be employed to illustrate the recomposition of white light, shows that whenever, by the action of two similar prisms *of the same material*, dispersion is destroyed the deviation is also destroyed. Sir Isaac Newton, after some research in this direction, came to the conclusion that this result was true generally, whether the prisms were of the same material or not, and that it was impossible to obtain deviation without dispersion, or dispersion without deviation. This conclusion we now know to be wrong, for let us see what it leads to. Let d and D denote respectively the extreme dispersion and mean deviation produced by P, and d' and D′ the dispersion and deviation produced by P′, then if, on refraction through the combined prisms in the way explained above, the dispersion and deviation are simultaneously destroyed, we have—

$$d = d' \text{ and } D = D',$$

and therefore—

$$\frac{d}{D} = \frac{d'}{D'},$$

$$i.e.\ \omega = \omega'.$$

If, then, Newton's conclusions were true generally, it would mean that all media have the same dispersive power. Experiment has shown that this is not the case (Art. 97).

Since the refractive indices and dispersive powers are different for different substances it is possible to make combinations of prisms which will either (*a*) deviate but not disperse, (*b*) disperse but not deviate.

(2) Let the prisms C, F (Fig. 123), made of media of different dispersive powers, but of equal refracting angles, a, yield spectra $r\,v$, $r_1\,v_1$ from a beam of white light. Let D and D_1 be the middle of these spectra. If F have the greater dispersive power $r\,v$ is not as long nor as deviated as $r_1\,v_1$. It is possible now to cut down the angle of F till its spectrum is of the same length as that of C. Let F_1 be the new prism yielding the spectrum $r_2\,v_2$, equal in length to $r\,v$; $O_2\,D_2$ is now less than $O\,D$; hence, on combining C and F_1, as in (4), we get a combination which deviates, but does not disperse. The deviation

$$O_3\,D_3 = O\,D - O_2\,D_2.$$

This fact was discovered by Hall in 1730, and first used by Dollond, a London optician, in 1757.

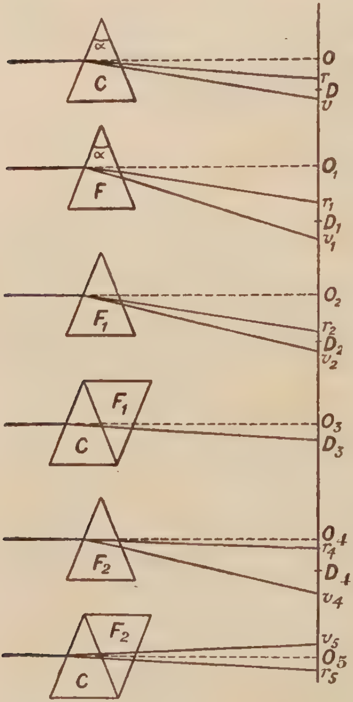

Fig. 123.

Example. Suppose that C and F are made of crown and flint glass respectively, their refracting angles being equal and small. $r_1\,v_1$ is approximately twice $r\,v$, and if F be cut down to F_1 to make $r_2\,v_2$ equal $r\,v$ the combination of the fourth figure deviates but does not disperse, the deviation amounting to about one fifth of the refracting angle of C.

(3) Instead of cutting down the angle of F, as described above, we may cut it down until the mean deviation which it produces is equal to that produced by C. This gives us F_2, where $O_4 D_4 = O D$; $r_4 v_4$ is still greater than $r v$, and hence on combining F_2 with C we get (6), which disperses but does not deviate. The dispersion $r_5 v_5 = r_4 v_4 - rv$. That is, the beam of light after emergence from the combined prisms will continue approximately in its original direction, but the beam itself will be dispersed, and, if allowed to fall on a screen, will show a spectrum. This is the principle of the *direct-vision spectroscope* (Art. **146**).

Example. With crown and flint glass prisms of small angle the angle of F_2 would be about nine-tenths that of C and the dispersion produced is about one seventeenth of the refracting angle of C. It is also in an opposite direction to that produced by C.

99. Dispersion in a lens. When a pencil of compound light is refracted through a lens, it suffers dispersion just as in refraction through a prism. Thus, if a diverging pencil of solar light, P *a b* (Fig. 124), be incident on the convex lens L, then the red rays, being the least refrangible, are brought to a focus at R, while the violet rays converge to a focus V nearer the lens. The orange, yellow, green, and blue rays converge to points intermediate between R and V, and thus, instead of the refracted rays all meeting in one focus, the rays of each colour converge to their own focus, and the image formed on a screen placed anywhere near V or R will be coloured at its edges. If the screen be placed anywhere near V, between the line *vr* and the lens, then the outer edge of the image will be red, but,

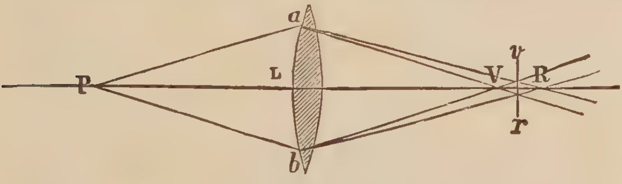

Fig. 124.

if placed beyond *vr*, then the outer edge shows violet. This fact is taken advantage of in focussing an image on a screen. The points V and R are very close together, and the best *definition* of the image is obtained when the screen is at *vr*. This adjustment is readily made by gradually changing the position of the screen until the colour showing at the outer edge of the image changes from red to violet. When this change of colour takes place the screen is in the position indicated by the line *vr*.

This effect of the dispersion of light when refracted through a lens is called **chromatic aberration** and was a great source of trouble in the construction of optical instruments, until it was shown that it was possible to obtain deviation without dispersion; that is, that it was possible to make the rays converge to a focus without obtaining a coloured image. This result, as in the case of prisms, is achieved by combining two lenses of different dispersive powers, and such that the chromatic aberrations which they singly produce are equal and opposite. For example, if a convex lens of crown glass (focal length, 20 cms.) be combined with a concave lens of flint glass (focal length, 34 cms.), the combination is equivalent to a convex lens of about 49 cms. focal length, and the image produced by it is almost entirely free from all colour defects. Such a combination is said to be *achromatic*.

100. The prismatic spectrum. The *prismatic* spectrum is the spectrum obtained by the decomposition of white light on refraction through a prism.

All radiant waves are capable of refraction and dispersion, and thus, when a beam of white light is refracted through a prism, the emergent beam is made up of a series of rays, separated and arranged in order of *continuously* increasing refrangibility. Beginning at the less refrangible end of the spectrum determined by this emergent beam, and travelling in the direction of increasing refrangibility, we pass a group of rays known as the *dark heat rays* which do not excite the sensation of sight. Then we come to another group, the rays of the *visible spectrum* ranging through the colours red, orange, yellow, green, blue, and violet. This group of rays, in addition to possessing heating properties, has the peculiar property of exciting the optic nerve, and thus producing the sensation of sight. In this visible spectrum the intensity of the light is different in different parts, being a maximum in the yellow and gradually diminishing on both sides towards the red and violet, as shown by the ordinates of the *intensity curve* of Fig. 125. Beyond the visible spectrum we come to the *dark chemical rays* or *actinic rays.* These extend for a considerable distance beyond the violet, and are characterised by their power of producing chemical action in a certain class of substances.

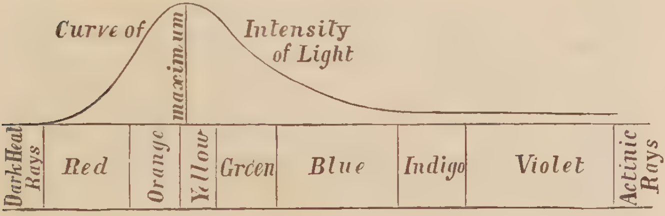

Fig. 125.

The complete spectrum may thus, at first, be regarded as made up of *dark heat*, *luminous*, and *actinic rays.* The only essential difference between these is that of wave-length, which *continuously* decreases from the first to the last.

101. The invisible parts of the spectrum. (1) *The actinic or ultra-violet rays.*—If an ordinary photographic plate be exposed in the prismatic spectrum it will be found on development that the red light has had little effect upon the plate, which is, however, strongly affected by the blue and violet light * and also to a large extent by the rays beyond the violet, these *ultra-violet* rays being particularly able to decompose the sensitive silver salt (silver bromide), though not able to excite the sensation of vision. Glass prisms and lenses absorb ultra-violet light to a great extent, but if quartz prisms and lenses be used it will be found that the ultra-violet portion of the spectrum of an incandescent body extends to an enormous extent beyond the violet end of the visible spectrum.

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* In ordinary photographic work the inability of the ordinary plate to record colours in their correct luminosities is a serious drawback in all work other than that of mere black-and-white. Special plates called orthochromatic have, however, been manufactured which, in conjunction with a specially prepared coloured screen, nearly reproduce the correct luminosities.

(2) *The dark heat or infra-red rays.*—In 1810 Herschel found that as he moved a small thermometer through the solar spectrum from violet to red, it showed only a little rise of temperature in the blue end of the spectrum, a little more in the green, a large rise in the red end, and even for some distance below the red end the thermometer was very sensibly heated. Since then much work has been done on the infra-red portion of the spectrum. As glass absorbs these dark heat rays, prisms and lenses made of rock salt or fluorspar are used instead, and the thermometer is replaced by a lamp-blacked linear thermopile* or bolometer.† Lampblack absorbs all the radiation which falls on it, hence the energy measured at any point is the *total* energy sent to that portion of the spectrum.

Exp. 52. Send an electric current of gradually increasing strength through a platinum wire in a dark room. The wire soon becomes just perceptibly warm to the touch, then too hot for the hand to bear, and soon so hot that the heat radiated from it may be felt at a distance of several inches. But it is still invisible. All the rays yet emitted are obscure rays. After a time, as the temperature increases, the wire becomes faintly visible, first as a peculiar flickering "grey glow," followed as the temperature rises by an emission of the extreme red rays of the spectrum. With further rise of temperature the yellow and orange rays are added to the red and obscure rays, then follow the green, blue, and violet rays. The wire being then white hot and very near its melting-point the experiment must come to an end. This, considered in the light of the experiment with the thermometer in the various spectrum rays, shows that the varying thermal and luminous effects depend upon the frequency or wavelength of the emitted rays, and that the greatest heating effects are due to waves of low frequency and comparatively great length.

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* See *Higher Text-book of Heat*, § 97; *Intermediate Text-book of Magnetism and Electricity*, § 194.

† See *Higher Text-book of Heat*, § 109.

It is thus possible to estimate the temperature of an incandescent body, the light of which is due to heat, by carefully noting its colour. As a body is heated the first colour, a dull red, appears at 525° C. This turns to cherry colour at 800° C., and to a bright cherry at 1000° C. Bright orange appears at 1200° C., white at 1300° C., and dazzling white at 1500° C. and above.

102. Transmission and absorption of radiation. Experiments show that when radiation of a definite kind is transmitted through a substance the amount transmitted decreases in geometrical progression as the thickness increases in arithmetical progression—that is, each layer of the substance, of a given thickness, transmits the same proportion of the radiation which enters it. Thus, if I_0 denote the quantity of radiation entering the substance, the amount which emerges after traversing unit thickness is I_0a, where a is a constant. Similarly, on transmission through another layer of unit thickness it is reduced to I_0a^2 and therefore, after transmission through a layer of thickness n, the quantity of transmitted radiation is given by—

$$I = I_0a^n.$$

The constant a has been called the *coefficient of transmission*. It is independent of the intensity of the incident beam and depends only on the nature of the substance and the wave-length of the radiation employed. When radiation of a compound nature is transmitted by any substance, its various constituents are absorbed in different degrees, and thus the nature of the transmitted radiation is subject to continuous change. The character of this change is, however, such that the nature of the transmitted radiation tends to become constant and capable of further transmission without absorption. For this reason radiation which has passed through a thick plate of any substance passes readily with little loss through another plate of the same substance.

Absorption, then, is the prime factor in the production of colour. If white light falls upon a plate which absorbs unequally the rays of different wave-length, the emergent light will be coloured. For considerable thicknesses the colour remains the same for different thicknesses, the shade becoming darker, but with thin enough layers the colour gradually alters. Thus thin plates of cobalt glass transmit chiefly blue light, while thick plates transmit a preponderance of red.

The light reflected from a body is also very often coloured. This is often due to the reflection not being wholly superficial. A portion of the incident light penetrates the body for some distance, suffers internal reflection and returns to the front surface, from which it emerges in line with the reflected light. This portion has undergone absorption and hence is usually coloured.

Exp. 53. Admit a beam of white light into a dark room. Reflect it by a coloured surface and catch the reflected beam on a white screen. Observe that this screen now appears of the same colour as the reflecting surface. Note also the reflections of coloured shop-signs.

A large crystal of sulphate of copper is transparent and deep blue, because it absorbs all but the blue components of white light before the light has travelled very far through it. But when the crystal is crushed to a fine powder, not only does it become opaque, but its deep colour is reduced to a pale Cambridge blue. The light cannot then penetrate far enough into the crushed mass for any great absorption to take place. So it is with other coloured crystals. When crushed they assume a much paler tint.

103. Colours of bodies. If a piece of red cloth or a red poppy be held in the red part of the spectrum it appears red. Held in the green or blue part of the spectrum it appears black. So, a green leaf is distinctly green in the green portion of the spectrum, but is black elsewhere. Similarly a piece of cloth exhibits the colour which it has in the sunlight only at that part of the spectrum which is coloured like itself.

These experiments show that a body which is red in daylight is able to reflect red rays only. As it appears dark in the green or blue light of the spectrum, it reflects no green or blue rays. The same reasoning applies to other colours ; a green surface reflects only green, blue only blue rays, and so on.

We now understand, with the help of Art. 102, the meaning of the colours which bodies are seen to exhibit in white light. White light is made up of many colours. When it falls upon a red surface, only that part of it which is red is reflected. The other spectrum colours are absorbed by the surface. When a body appears yellow, we are to understand that all the colours except yellow are absorbed.

So far we have assumed that the colours of bodies are simple, and not made up of a mixture of two or more different colours. It is difficult to obtain a pure green or a pure yellow or blue, and hence it often happens that when a coloured cloth is held in other parts of the spectrum than that which matches its colour, it appears coloured. A piece of green baize appears bluish in a blue light and yellowish in a yellow light, because most green objects transmit also some blue and yellow light.

If a piece of cloth of different tints is looked at in a light which is deficient in one or more of the colours of the spectrum, or in a light where one of the colours predominates, it does not always *look* the same as when viewed in daylight. If the light lacks one of the tints which the cloth exhibits in daylight, then that particular tint cannot be reflected by the cloth. In gas light yellows are brightest, because a gas-flame is chiefly a yellow light. But since a white body seen by yellow light appears yellow, the difference between white and yellow by gas light is much less distinct than by the white light of day.

We have now no difficulty in explaining the colours of transparent plates. A plate of red glass lets only red rays pass through, a green plate transmits the green rays, and so forth. A blue plate does not allow red rays to pass through it. Hence, if a beam of sunlight be made to fall upon a piece of red glass, and if a piece of blue glass be held in the course of the red light, it follows that as none of the red light can pass through the blue glass, the two plates together cut off all the light, and the source of light is invisible; in practice, however, certain rays may get through, in which case the source will be visible but dark in colour.

Exp. 54. Look at a gas flame through two plates of different colours, say blue and red, red and green, yellow and blue. The flame is invisible or nearly so, if the tints are sufficiently deep. It is difficult to get glasses of pure colour, so that ordinary red glass allows some other rays than red to pass through. Some of these may be able to pass through the second piece of glass.

Observe that a white cloth appears red in a red light, because it reflects red, blue, green, or any other colour. In fact, it appears white in daylight because it reflects all the colours.

104. Primary and complementary colours. A *primary* colour is defined as that which cannot be imitated to the eye by the mixture of any other colours. Maxwell showed that there are three primary colours—red, green, and violet—and that any other colour can be formed by mixing suitable proportions of these.

Colour mixtures may be effected in the following way: A spectrum is thrown upon a screen which is provided with adjustable slits, which can be opened to various extents and also shifted to occupy different places in the spectrum. In this way beams of different colours are let through, and these can be combined by a judicious arrangement of mirrors and lenses. **Newton's disc (Art. 106)** affords another method.

Any two colours which produce the sensation of white when mixed are said to be *complementary*.

If we open a slit in the yellow and move another slit up and down the spectrum, we find that when it is in the blue the mixture of the two beams produces white. Thus yellow and blue are complementary, so also are red and greenish-blue, and green and purple. If, however, we mix yellow and blue pigments, the result is not white, but green. Neither the blue nor the yellow pigment is as pure as in the spectrum. There is green in both. The colour of the mixture is *chiefly* due to light which *both transmit,* which in this case is the green. In water colours the light reaching the white paper through yellow and blue paint is mainly green, and this being reflected from the paper gives the green colour.

Exp. 55. Throw a continuous spectrum upon a screen. Cut away part of the screen, so that all the colours except red pass through. By means of a reversed prism (Art. 98, 1) and lens combine all the colours that pass through the opening. Then the combined colours form a colour—greenish-blue—exactly complementary to the red.

105. Theories of colour vision. Colour blindness. The commonly accepted theory of colour vision is that due to Young and Helmholtz, and it postulates that, just as there are three primary colours, so we have three sets of nerves by which colours are appreciated. The three colours to which these nerves respond are the red, green, and blue. It is to be noted that these are not the three primary colours. Red, green, and blue are therefore the three fundamental sensations, and the colour of a body simply depends upon the proportion in which the three sets of nerves are excited. Fig. 126—due to Captain Abney—shows that each region of the spectrum exercises all the sensations to varying degrees, and if we find a number of points in the spectrum such that the sum of the red-sensation ordinates is equal to the sum of the green-sensation ordinates, and also equal to the sum of the blue-sensation ordinates, the combination of the colours at these points would result in white light.

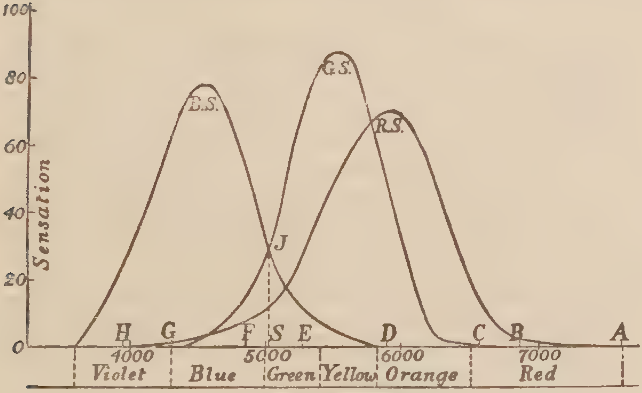

Fig. 126.

If one set of nerves is absent or inactive, the person is said to be colour-blind. Suppose, for example, a man is red-blind (this is the commonest form of colour-blindness), then he has no sensation corresponding to the curve RS (Fig. 126), and therefore the red constituent of all colours is unappreciated. To such an observer the point S in the spectrum appears white, for the ordinate JS is the same both for blue and green sensations, and, according to him, the spectrum is composed of two colours, yellow and blue, and other colours which are more or less shades of these. Red to him is only dark yellow, and very often the spectrum is considerably shortened at the red end. Green is rather muddy yellow, and the middle of the spectrum (around S) is white or grey, while the violet he prefers to call dark blue. Of course colour-blind persons learn by experience to give many colours their proper names, but in some occupations, such as engine-driving and signalling, colour-blindness is such a great defect that candidates for such posts are carefully selected by tests both with the spectrum and with coloured skeins of wool.

106. Recomposition of white light. The experiments we considered in **Art. 95** were analytical. We have determined the composition of white light by decomposing it into its constituent coloured rays. But there are many ways by which we can reverse the process, that is, we may start with the separate colours, and recombine them into a beam of white light, thus effecting a synthesis:

1. By the use of a second prism exactly like the first, but with its refracting edge turned in the opposite direction, as in Fig. 122.

2. By receiving the spectrum on a line of plane mirrors so that a separate colour falls on each, and then inclining the mirrors so that all the coloured rays are reflected to the same spot on a screen.

3. By interposing in the course of the spectrum rays an achromatic lens. A cylindrical lens with its geometric axis parallel with the slit gives the best results.

4. By **Newton's disc.** This is a circular disc of cardboard divided into sectors painted with the colours of the spectrum and attached to a whirling table. When the disc is rapidly rotated it appears nearly white, not because there is any real mixture of colours, but because of the fact that luminous impressions on the retina of the eye persist for a small fraction (about $\frac{1}{10}$) of a second, so that before the impression due to any one colour has died away it is

succeeded by all the other colours. Thus there is a physiological though not a physical blending of the colours.

5. By combining complementary colours as in Art. **104**.

107. **The rainbow.** When the sun is shining upon a cloud of rain or on the spray from a waterfall or fountain, an observer standing with his back to the sun, and facing the rain or spray, often sees a circular arc of colour apparently in the midst of the water drops. Such an arc is called a rainbow, and is due to the reflection and refraction of the light falling upon the drops of water.

Lunar rainbows are also occasionally seen. Owing to their faintness they usually appear destitute of colour.

The large circular rings or *Halos* which are sometimes seen around the moon, and more rarely around the sun, are due to refraction through tiny hexagonal ice crystals. The smaller brightly coloured circles or *Coronas* seen close round the moon, especially when it is full, are due to diffraction effects produced by very small water drops in high clouds; their formation lies beyond the scope of the present book.

108. The scattering of light by very fine particles.* It can be proved experimentally in the case of sound waves and mathematically for all wave motion that the smaller the wave-length of any radiation the more perfectly will an obstacle stop the waves and scatter them. Red light has a wave-length double that of violet light; hence when white light is travelling through a medium containing very fine particles in suspension we should expect the scattered light to have a blue tint and the transmitted light a red tint. This is borne out by the facts that street lamps look very red in a fog, and the setting sun appears redder as it approaches the horizon. Also the smoke from a lighted cigarette or wood fire, a distant haze, and a reservoir of water containing very fine particles in suspension look blue. The colour of the sky may be explained in like manner, the scattering in this case being due either to fine salt particles, to fine metallic dust from meteorites, or to the gaseous molecules themselves.

A pretty experiment to illustrate the colour of the setting sun can be performed by illuminating a screen by a parallel beam of white light from an optical lantern, and then inserting in the path of the beam a glass cell containing a freshly-made dilute solution of sodium thiosulphate—the "hypo" of the photographers—to which a little dilute hydrochloric acid has been added. Precipitation of sulphur gradually occurs, and the image on the screen gradually changes colour to orange and to red, and finally is obscured.

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* See also Art. 142.

EXERCISES VIII.

1. Draw the section of a prism. Draw also the section of a beam of sunlight passing through the prism, and show by your sketch how this light is acted on by the prism.

2. Given a powerful source of light, such as a lime-light or an electric-light, explain how you could obtain a spectrum of it on a screen.

3. Describe an experiment proving that white light is compound. How can it be shown that the constituents into which it is resolved are not likewise compound?

4. A spectrum cast upon a white screen is looked at through a red glass. What appearance does it present, and what is the cause of this appearance?

5. How would you disprove experimentally the assertion that white light passing through a piece of coloured glass acquires colour from the glass? What is it that really happens?

6. If you hold one piece of glass up to the sun it appears dark red; if you hold another up to the sun it appears dark blue. If you put the two glasses together you cannot see the sun through them at all. How is this?

7. A lamp-frame, looked at through a glass prism, appears to be coloured blue on one side, and red on the other. Draw a picture tracing the rays from the lamp to the eye, and showing which side of the coloured image is red and which side is blue.

EXAMINATION QUESTIONS.

1. Describe an experiment to show that white light is of a composite character.

2. What details must be attended to in order to obtain a pure spectrum on a screen?

3. A white object on a black ground is seen through a magnifying lens, and appears surrounded by a coloured fringe. Explain the principles on which this depends. How are lenses constructed which do not give rise to these coloured fringes?

4. The bar of a window is viewed through a glass prism held so that its edge is parallel to the bar. Describe and explain the appearances presented.

5. Light emanating from a luminous point in a dark room with white walls passes through a convex lens and forms an image of the point upon the wall. A prism is then interposed close to the lens, between the lens and the wall. Describe what you would see happen, and draw a diagram to show the paths of the rays of light which produce the effect that you see.

6. What is meant by *minimum deviation ?*

White light passes through a vertical slit placed parallel to the refracting edge of a glass prism. Draw a diagram showing the slit and the prism as seen from above and also the paths of the red and blue rays from the slit which suffer minimum deviation in passing through the prism.

Has every prism a position of minimum deviation for light of given colour ?

7. A wire is heated white hot, and an image of it is thrown on a screen by means of a lens. Draw a figure showing how you would arrange a prism in the course of the rays, and where you would place the screen to obtain a good spectrum, and indicate on your figure the positions of the different colours.

8. A glass prism is held in front of the eye with its edge vertical and to the right, and the flame of a spirit lamp on which some common salt has been sprinkled is viewed through the prism. Where will the flame appear to be ?

Illustrate your answer by a diagram and say what will be the effect of substituting the flame of a candle for that of the spirit lamp.

9. A horizontal slit in a shutter, through which light is passing, is looked at through a prism, the thick part of which is downwards, by an observer who is below the slit : describe the appearance seen, and give a diagram showing the appearance of the slit to the observer.

10. Describe the arrangement of apparatus you would employ to project the spectrum of an electric arc light upon a screen. Draw a diagram of the apparatus, and trace carefully the courses of the red and violet rays.

11. A ray of mixed red and blue light is incident on a prism of about 30°, the plane of incidence being perpendicular to the refracting edge. The prism is first held so that the ray falls normally on the first surface ; then it is held so that the ray makes a considerable angle with the normal. Draw figures showing what happens in each case.

12. A glass prism is set up vertically with its refracting edge in front of, and at a moderate distance from, a narrow slit illuminated by red light, and behind the prism, at a moderate distance, is set a vertical screen.

Sketch and explain the passage of the light from the slit through the prism to the screen (1) when the prism only is between the slit and the screen, (2) when, in addition, a convex lens is placed between the slit and the prism at a distance equal to its focal length from the slit and another convex lens between the prism and the screen at a distance equal to its focal length from the screen. What differences would you observe if the slit were illuminated (1) with blue light, (2) with white light?

13. Define "dispersive power." Explain how to combine prisms so as to produce (*a*) deviation without dispersion, (*b*) dispersion without deviation of a given mean ray.

14. Show that it is possible to make an achromatic prism with two sorts of glass.

15. By what experiments would you show that the radiation from an arc lamp extends beyond red at one end, and beyond violet at the other end of the spectrum? In what respect do these invisible radiations differ from the visible radiations?

16. What evidence is there that radiant heat is physically of the same nature as light?

17. How would you arrange an experiment to determine the percentage of light that is transmitted through a neutral tinted glass plate?

If a plate of such glass allowed 40 per cent. of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming no light to be lost by reflection at the surfaces in either case?

18. A snow-covered landscape when viewed through a piece of red glass appears red. How does the glass produce such a result? How would you verify your explanation by experiment?

Red light is found to penetrate a fog more effectively than light of any other colour. This being so, would the penetrating power of an arc lamp be increased by passing its rays through red glass?

19. A round stick 1 inch in diameter is placed vertically with its centre 6 inches in front of a vertical screen. At a distance of 1 foot from the screen two sources of light, one blue and the other yellow, are so adjusted that the two shadows of the rod due to them overlap to the extent of $\frac{1}{2}$ inch. Draw a diagram, approximately to scale, of the shadows on the screen, and describe the colour effects visible on it.

20. To what are the colours of objects as ordinarily seen due—for instance, red glass, blood, red cloth, red copper?

CHAPTER X.

VELOCITY OF LIGHT.

109. The velocity with which light travels through any medium is inconceivably great, but varies with the nature of the medium. The velocity *in vacuo* is taken as *the* velocity of light, and the velocity in any other medium may then be determined, as explained in Art. **50**, from the absolute refractive index of that medium. For, if V denote the velocity of light in *vacuo*, and V_m its velocity in any given medium, then—

$$\frac{V}{V_m} = \mu,$$

where μ denotes the absolute refractive index of the medium.

Hence, if we can determine the velocity of light in any medium, such as air, we can calculate its velocity in any other medium, or *in vacuo*.

The velocity of light has been determined in two general ways :—

(1) From observations of celestial phenomena.

(2) By direct terrestrial experiments.

The first gives the velocity *in vacuo*, the second the velocity in air.

110. Determination of the velocity of light from observations of celestial phenomena. The first computation of the velocity of light by this method was made, in 1675, by **Roemer**, a Danish astronomer. He deduced his result from observations of the eclipses of Jupiter's first satellite, Io. This satellite is eclipsed to us once during each revolution when it passes behind the planet into the shadow cast by the sun. This occurs at intervals of about 42 hours. The instant at which the eclipse should take place can be accurately calculated from dynamical considerations based upon the mean of a large number of observations. Roemer timed the eclipse when the earth was in that part of its orbit nearest to Jupiter, and from this time calculated the times of the eclipses which would occur throughout the year. During the succeeding months he set himself to observe these eclipses and he found that the observed times were always later than the calculated times, and also that the difference between these two times varied with the relative position of the earth and Jupiter. From a careful analysis of the observations it was found that the difference between the observed and calculated times, increased as the earth moved away from Jupiter, reached a maximum about six-elevenths of a year afterwards, when the distance between these two bodies had attained its greatest value, and gradually decreased again to zero in another six-elevenths of a year when the earth was again in a position nearest to Jupiter. From this it is evident that the interval between the actual occurrence of the eclipse and the instant of its observation on the earth is equal to the time taken by light in travelling from Jupiter, or rather from Jupiter's satellite to the earth.

Let S (Fig. 127) represent the sun, E_1, E_2, E_3 the orbit of the earth, and J_1, J_2, J_3 that of Jupiter. Both earth and Jupiter move round the sun in the same direction; the times of revolution being one and twelve years respectively. Starting with the planets in conjunction at E_1, J_1, they will be in opposition at E_2, J_2 six-elevenths of a year later and again in conjunction at E_3, J_3 twelve-elevenths of a year after the previous conjunction, and it is now evident that as the earth moves to E_2 and Jupiter to J_2 the observed times of the eclipse lag behind the calculated times, the lag being a maximum at E_2, J_2. As the motion still ensues the lag decreases, and by the time the earth and Jupiter have reached to the positions E_3, J_3 the observed and calculated times once more agree. It is also evident

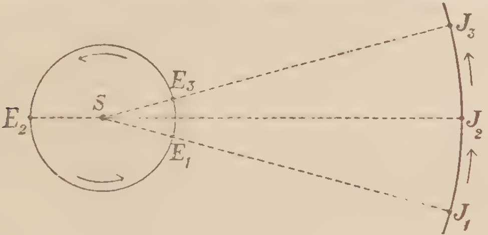

Fig. 127.

that the maximum difference between the calculated and observed times is equal to the difference in the times taken by light in travelling the distances J_1 E_1 and J_2 E_2, *i.e.* equal to the time taken in travelling a distance equal to the diameter of the earth's orbit.

The diameter of the earth's orbit is about 185,600,000 miles. The eclipse at E_2 is always about 16·5 minutes later than the time calculated from observations at E_1. Hence we have—

$$\text{Velocity of light} = \frac{185{,}600{,}000}{16{\cdot}5 \times 60} \text{ miles per second.}$$

This gives a velocity of about 187,000 miles per second.

About fifty years after the time of Roemer, Bradley, the English astronomer, gave an explanation of the phenomenon of *astronomical aberration*, based on the fact that light travels through space with a definite velocity. This phenomenon is due to the fact that both the earth and light travel through space with definite velocities, and hence the direction in which light from a star reaches the earth will be in the direction of the velocity of the light *relative to the earth*. Thus, if A (Fig. 128) represent the position of the earth when light from a star, S,* starts from S′, and if the velocities of the earth and of light be such that the former travels from A to B while the latter travels from S′ to B, then the direction in which the star is seen from the earth is parallel to A S′, and not to the true direction, A S″.

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* Many of the stars are at such great distances from the earth that, neglecting aberration, they are apparently seen in the same direction whatever the position of the earth in its orbit. The distance of the nearest fixed star is greater than 200,000 times the distance of the sun.

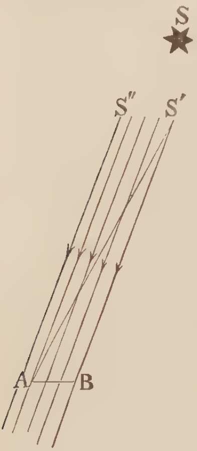

Fig. 128.

From our construction it is evident that—

$$\frac{\text{A B}}{\text{B S}'} = \frac{\text{Velocity of the earth}}{\text{Velocity of light}}$$

and, when the angle S″ A B is a right angle, that is, when the true direction of the star is at right angles to that in which the earth is moving in its orbit, we have—

$$\frac{\text{A B}}{\text{B S}'} = \tan \text{B S}' \text{A} = \tan \text{S}' \text{A S}'',$$

and the angle S′ A S″ is called the *aberration* of the star.

Hence, if V denote the velocity of light, and v the velocity of the earth in its orbit, we get—

$$\frac{v}{V} = \tan \theta,$$

where θ denotes the aberration of the star.

Of the quantities involved in this relation v and θ can be determined by astronomical observation, and V can then be calculated.

Aberration is perhaps more clearly understood if we consider a man running through a shower of rain falling vertically. The drops will strike him in the face (or he will strike his face against the drops), and the rain will therefore appear to him to come from a point not straight above him but somewhat in front. The effect depends entirely on the velocity with which the rain is falling *when it reaches him*, not on how long it has been falling. So the displacement of a star by aberration is the same for all stars, and quite independent of their distances. As the star is never seen in its true position the angle cannot be measured direct. The angle measured is the angle between the apparent positions when the earth moves in opposite directions. This of course is double the aberration, and has been found to be 20·44 seconds.

The value obtained for V by this method is about 185,000 miles per second.

111. Direct determination of the velocity of light by terrestrial experiment. Two distinct methods have been devised to determine the velocity of light by direct experiment.

I. Fizeau's method. The principle of this method is simple. Let S (Fig. 129) represent a source of light and M a plane mirror. Now if a ray of light, S M, be incident normally on the mirror M, it will be reflected back along M S, and an observer behind S will see an image of S in the mirror. But, if a toothed wheel, having the teeth and spaces of equal width, be interposed at W in the position indicated in the figure, it may be rotated at such a rate that the light incident through any space will, after reflexion, be received on the back of the next tooth, and thus no image of S will be seen in the mirror. When this is the case it is evident that during the time taken by the wheel to rotate through the angular width of one of the spaces, light travels from W to M and back again. Hence, to determine the velocity of light, from this experiment we have that—

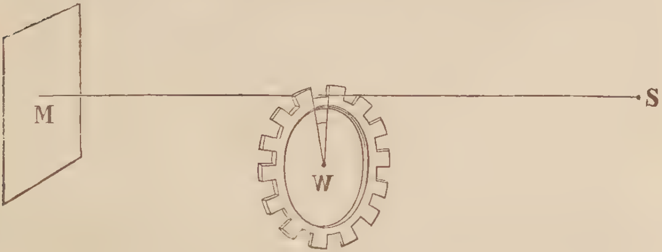

Fig. 129.

$$V = \frac{2\,W\,M}{t},$$

where t denotes the time in which the wheel rotates through the angle subtended by one of the spaces, at the centre of the wheel.

It is further evident that if the wheel be rotated at twice the above rate the reflected ray will pass through the next space, and the image will again become visible, and if rotated at treble the rate extinction again takes place, and so on.

In the application of this principle Fizeau employed somewhat complicated apparatus, the essential parts of which are shown in Fig. 130. A source of light, S, was placed so as to send a beam of light down the side tube *t* on the mirror *m*, made of unsilvered glass. This mirror is inclined at an angle of 45° to the axis of *t*, and reflects the light along the main tube T on to the lens L. The position of this lens is so adjusted that the rays emerge parallel, and after traversing a distance of about 5½ miles, fall on the lens L′, which causes them to converge through the tube T′ on to the mirror M, from which they are reflected back along the path by which they came. On reaching the mirror *m* the light is partially reflected to S, but a portion passes through and reaches the observer's eye at E, after passing through the lenses *l* and *l′*, which are adjusted to give distinct vision of the image.

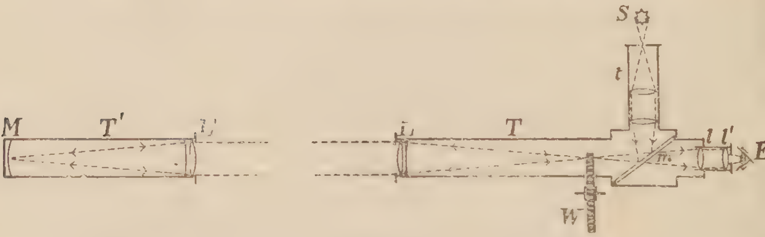

Fig. 130.

The wheel, placed at W, is driven by clockwork, and by adjusting its rate of rotation the image can be made to disappear and reappear successively several times. The wheel employed by Fizeau had 720 teeth and 720 spaces, the width of the latter being equal to that of the former. The distance W M was about 8,663 metres, and the first eclipse of the image took place when the wheel revolved 12·6 times per second. Hence, the time taken by the wheel to rotate through the angle subtended by one of the spaces is $\frac{1}{2 \times 720 \times 12{\cdot}6}$ of one second. In this time light travels from W to M and back again, a distance of $2 \times 8{,}663 = 17{,}326$ metres. Therefore, for the velocity of light we have—

$$V = 17{,}326 \times 2 \times 720 \times 12{\cdot}6 = 314{,}000{,}000 \text{ metres per second.}$$

This result is about 195,000 miles per second, and is somewhat in excess of the result obtained by more recent experiments.

In 1876 M. Cornu carried out a careful determination by Fizeau's original method, but on a larger scale, the distance W M (Fig. 130) being 15 miles, and found the value of V to be 300,330,000 metres per second in air, corresponding to 300,400,000 metres per second in *vacuo*. This result is also too high, more recent work having shown it to be less than 300,000,000 metres per second.

II. Foucault's method. This method is somewhat more complicated both in theory and in practice. Adopted in 1850 it utilises the principle of the rotating mirror as first employed in 1834 by Wheatstone, to determine the duration of the electric spark. The principle of the method is as follows:—Solar light is transmitted through a narrow rectangular aperture *s* (Fig. 131), down the middle of which extends a vertical wire. The light proceeds through the achromatic lens L, falls obliquely upon a plane mirror *m*, and then comes to a focus at M. At M is placed a concave mirror whose centre of curvature is at *c*, the middle point

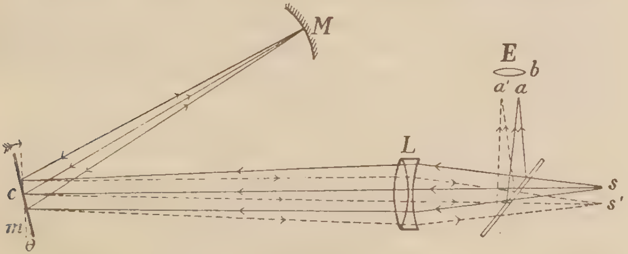

Fig. 131.

of *m*. For a certain position of *m*, a pencil of light, *s c*, starting from *s* is reflected from *m* to M (the central ray being incident along the normal *c* M), and then reflected back along the same path to *s*. For convenience of observation a thin parallel plate of glass is inserted between I and *s* at an angle of 45° to the axis of the central ray, so that the reflected beam is in part reflected and comes to a focus at *a*, which can be observed through an eyepiece, *b*.

If now the mirror *m* be made to revolve it will pass through the position just considered once in each revolution, and therefore an image of *s* will be seen, for an instant, once in each revolution. When the revolutions become sufficiently rapid (about 30 per second) these quickly succeeding images persist on the retina, and blend into one permanent image, still seen at *a*. When, however, the speed of rotation is greatly increased, the mirror, *m*, turns through an appreciable angle while the light is travelling from *c* to M and back again. For example, if the mirror turn through the angle θ while light travels from *c* to M and back to *c*, then the ray M *c* will not be reflected along *c s*, but along *c s'*, and the eye at E sees the image of *s* at *a'*. Hence, if we can determine the angle *s c s'* and the distance *c* M, we can calculate the velocity of light. For, by Art. **31**, the angle $s\,c\,s' = 2\,\theta$, and light travels a distance $2\,c\,\mathrm{M}$ during the time that the mirror revolves through an angle θ. If the mirror makes *n* revolutions per second, then the angular velocity per second is $2\,\pi\,n$, and the time in which the angle θ described is given by—

$$t = \frac{\theta}{2\,\pi\,n} \text{ seconds.}$$

Therefore, if *c* M be denoted by *l*, the velocity of light is given by—

$$\mathrm{V} = \frac{2l}{t} = \frac{4\,\pi\,n\,l}{\theta}.$$

Of the quantities involved in this relation, *n* and *l* are readily determined, and θ is equal to $\frac{1}{2}\,(s\,c\,s')$. In practice it would be very difficult to measure *s c s'* with any accuracy, but no difficulty is incurred in an accurate measurement of *a a'* which equals *s s'*, and θ is then evaluated in terms of the several distances involved. These distances are easily measured, and thus V can be calculated.

In the actual experiment the distance *c* M was 20 metres, the mirror was a piece of silvered glass, and was rotated by means of a small air turbine. The deflection *a a′* only amounted to 0·7 mm., but by means of the micrometer eyepiece this could be read to an accuracy of 1 in 150. The result finally obtained by Foucault was 298,000,000 metres per second.

In 1880 **Professor A. A. Michelson,** of the United States Navy, introduced great improvements in Foucault's method, the chief being the transference of the lens *L* from its position in Fig. 131 to a position between *c* and M. In this way the distance *c* M could be greatly increased (a distance of 2000 feet was attained) without any diminution in the brightness of the image. A deflection of 133 mm. was obtained, the plate of glass and micrometer eyepiece could thus be discarded and *s s′* directly measured. The rotating mirror was driven by an air turbine under perfect control, and its speed was measured by an electrically driven vibrating tuning-fork. His final result is 299,882,000 ± 60,000 metres per second.

Taking the mean of the best determinations we get the velocity of light *in air* to be 299,890,000 ± 60,000 metres per second; this corresponds to a velocity *in vacuo* of 299,970,000 ± 60,000 metres per second. In English units the velocity in air is 186,350 ± 40 miles per second.

112. Connection between the velocity of light and the refractive index of the medium. If a long tube containing water or other transparent medium be placed between *c* and M (Fig. 131), the displacement, from *a* to *a′*, will be greater or less, according as the velocity of light in the given medium is less or greater than the velocity in air. Experiment shows that light travels more slowly through a dense than through a rare medium, that is, the greater the refractive index of the medium, the less is the velocity of light through it.

Foucault, who was the first to carry out the above experiment, did not succeed in measuring the ratio of the velocities. This was left for Michelson; he used a tube of water 3 metres long and found the ratio of the velocity of light in air to that in water to be 1·330. Experiments on other transparent bodies have yielded similar results.

The question has also arisen whether the velocity of light of various colours is the same in air (or *in vacuo*)? Evidence shows that the velocities are identical, for if not at an eclipse of a star or at its reappearance it would appear coloured, and the image in Foucault's experiment would be dragged out into a spectrum. In other media the velocity is greater for red rays than for violet rays; thus Michelson found that red light travelled about 5 per cent. faster than blue light in carbon bisulphide.

113. Theories of the nature of light. Many of the effects of light and many crude optical instruments were known to the ancients, but of the theory of optics they were wholly ignorant. Pythagoras (B.C. 540-510) and Plato maintained that vision was a threefold phenomenon. Their idea was that the eye sent out a stream of potency or divine fire which combined first with the light of the sun and then with the emanation from the third body, the second combination completing the action of vision. Aristotle, in B.C. 350, struck the right chord by maintaining that light was not a material emission from a source, but a mere quality of potentiality of a medium existing between our eyes and the body seen.

114. The emission or corpuscular theory of light. In more recent times Sir Isaac Newton (1642-1727) upheld the corpuscular or emission theory of light. Light was supposed to be a swarm of corpuscles ejected from a luminous body at a great speed, and these corpuscles on entering the eye excited the sensation of vision. In open space their motion is rectilinear, but near material surfaces the motion proceeds along curves, this being due to either a repulsion or attraction of the corpuscle by the matter in question, a property which only extends to a short distance from the surface.

115. The wave theory of light. This theory may be said to have begun with Aristotle, but it was not till Huyghens took it up in 1678 that it was enunciated in a scientific manner. Since then Fresnel, Young, and Stokes have finally established it.

It postulates (Art. 3) that there is spread throughout space an all-pervading medium or ether, and that transverse oscillations in this ether constitute thermal, luminous, and electrical radiations.

Many have been the properties with which the ether has been endowed. As we can neither hear, taste, see, smell, nor feel it, we can never be directly cognisant of its properties; in fact, we are only aware of its existence because the sciences of radiant heat, light, and electricity demand such a medium for the transmission of their effects.

A luminous body is supposed to set up transverse vibrations in the ether in its neighbourhood; this disturbance then travels out through the ether, and on entering the eye excites the sensation of vision. Since the propagation is attended with transverse wave motion the ether must possess properties akin to rigidity and density, and at different times it has been considered to be an incompressible fluid and an elastic solid.

For most purposes it is unnecessary to consider the properties of the ether; light phenomena as a rule can be explained by simply supposing either transverse waves of displacement or electromagnetic waves to be travelling through the ether. The properties of wave motion are treated fully in Text-books of Sound, so that in this place we shall not dwell on the mechanics of the problem.

116. Crucial test between the emission and undulatory theories. Let μ be the index of refraction between two media a and b; and $v_a . v_b$ the velocities of light in these media. The relation between the three quantities is given[*] by

$$\mu = \frac{v_b}{v_a} \text{ on the Emission theory.}$$

$$\mu = \frac{v_a}{v_b} \text{ on the Undulatory theory.}$$

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[*] See *Text-book of Light*, Second Edition, Arts. 131, 132

Thus in a medium, such as water, for which $\mu > 1$, the emission theory states that the velocity is greater than the velocity in air, while the undulatory theory postulates the reverse. Foucault decided this point (see Art. **111**) in favour of the undulatory theory, and although this does not prove the undulatory theory to be right, it certainly proves that the emission theory, as at present enunciated, is wrong.

These results were assumed in Art. 50.

117. The rectilinear propagation of light. This was a strong point with the supporters of the emission theory, for, granted that the corpuscles moved in straight lines until acted on by the surfaces of media, the theorem that light travels in straight lines is self-evident.

On the undulatory theory it proved at first a great stumbling block, for the opponents argued that as sound waves easily bent around corners so also would light waves. In the case of sound waves it is easily proved by experiment that the shorter the waves the more sharply defined are the sound shadows.* Now the wave-length of the sound waves caused by a tuning-fork of frequency 512 per second is about two feet, while the wave-length of the yellow or mean light of the spectrum is only $\frac{1}{44500}$ inch, hence we should expect, even on the wave theory, the shadows to be very distinct, but never *absolutely* sharp.

This latter result is borne out by the fact that the best shadows are always bordered by alternate light and dark fringes called *diffraction* bands. The propagation of light is thus only approximately rectilinear.

Exp. 56. Cut a very narrow slit in a postcard, and place it in front of a white light. Cut another slit in another postcard, and hold this up at some distance in front of the eye, so that the flame, slits, and eye are in alignment. Diffraction bands will now be clearly seen bordering the direct view of the flame.

These diffraction bands are due to the superposition of light waves which have traversed slightly different paths. Other instances occur when a candle is observed through a silk handkerchief, a street lamp through an umbrella, and the sun through half-closed eyelashes.

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* See *Matriculation Sound.*

The colours obtained when oil is poured on water, or a piece of steel is heated in the flame, are also due to the same cause. So also are the colours of soap-bubbles, mother-of-pearl, and cracks in crystals and blocks of ice.

EXAMINATION QUESTIONS.

1. How has the velocity of light been determined from observations of the eclipses of Jupiter's first satellite?

Assuming that Jupiter's first satellite revolves round the planet in a constant period of forty hours, that the velocity of the Earth in its orbit is 18 miles per second, and that of light is 187,000 miles per second, find the greatest and the least apparent intervals between successive eclipses.

2. Describe the method by which Fizeau investigated the velocity of light.

3. Describe Foucault's method of measuring the velocity of light by means of a rotating mirror. What is the effect of introducing a tube of water (with glass ends) between the rotating and fixed mirrors, and what relation is there between the velocity of light in a medium and its refractive index?

4. Explain carefully some method of measuring the velocity of light. How has it been shown that lights of different colours travel through air at very nearly the same rate?

5. Describe a method of measuring the velocity of light based upon the use of a revolving toothed wheel. The distance between the two stations being 9·3 miles and the number of teeth being a hundred, the rotation is started and gradually increased in speed. Find the number of rotations of the wheel in a second when the light reflected from the distant station has disappeared and reappeared ten times. The velocity of light may be taken as 186,000 miles per second.

6. Describe a method by which the velocity of light has been determined by a rapidly revolving mirror. Assuming that the distance between the two mirrors necessary is half a mile and that the velocity of light is 187,000 miles a second, find the number of revolutions per second necessary to cause a deflection of the reflected ray through one degree.

7. Point out the difference between a wave of sound, a wave of light, and a wave traversing a stretched string.

8. What is meant by the aberration of light? How did Bradley explain it? Mention some familiar phenomena which illustrate the principle of his explanation.

9. How has it been shown that light travels more rapidly in air than in water?

CHAPTER XI.

SIMPLE OPTICAL INSTRUMENTS.

118. Artificial horizon. The altitude* of a star is frequently determined by a method based on the laws of reflection. The accuracy of the results obtained by this method furnish an indirect but rigorous proof of the truth of these laws. A vertical divided circle, adjusted in a vertical plane, carries a telescope T T (Fig. 132) which can be rotated round an axis passing through the centre of the circle. In making an observation the telescope is first pointed to a particular star, and the reading on the circular scale, for this position of the telescope, is accurately noted. The telescope is then turned into the position T′ T′, so as to view the image of the star, formed by reflection from the *horizontal surface* of mercury contained in the vessel M. The reading of the scale corresponding to this position is again noted, and the difference between the two readings—that is, the angle S′ P′ R—gives twice the altitude of

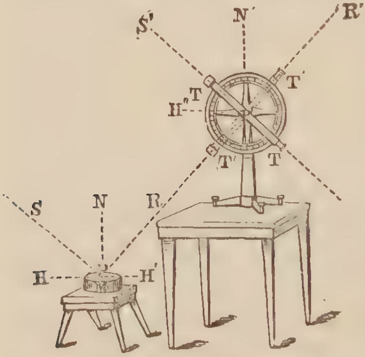

Fig. 132.

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* The altitude of a star is the angle between the direction of the star and its horizontal projection.

the star. For, *assuming the laws of reflection to be true*, we have—

$$S P N = N P R,$$

and therefore— $S P H = R P H'.$

But $R P' H'' = R P H'$ (Euc. i. 29);

therefore $R P' H'' = S P H,$

also— $S' P' H'' = S P H.$*

Therefore we have—

$$S' P' R = S' P' H'' + H'' P' R = 2 S P H.$$

But S P H is the altitude of the star; therefore S′ P′ R is twice the altitude of the star.

The accuracy of the results obtained by this method conclusively proves the truth of the laws of reflection.

119. Hadley's sextant. The sextant is an instrument employed for measuring the angle between two distant objects, as seen from the position occupied by the observer. The principle of its action has already been explained in Art. 31. The essential parts of the instrument are shown in Fig. 133. The frame is made up of the circular arc, S S′, and the two arms, S C and S′ C. These two arms, which are radii of the circle of which S S′ is an arc, intersect at C the centre of the circle, and C I is an index arm which can be rotated about an axis passing through C. Two plane mirrors, A and B, are attached to this arrangement; A is fixed on the arm S′ C, and B is attached, at C, on the index arm C I; both mirrors being

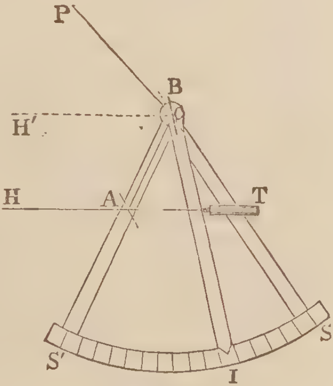

Fig. 133.

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* S P and S′ P′ are parallel. Compare footnote to Page 205.

perpendicular to the plane of the paper. The mirror A is unsilvered, or only partially silvered, so that an observer looking through the telescope, T, which is directed towards A, can see objects in the direction T H. When I is at S, the planes of the mirrors, A and B, are parallel, so that any ray H′ C, incident on B parallel to H T, is reflected along C A to A, and thence, along A T, to T.

The observer looking through T thus sees objects in the direction T H (or C H′), both directly through A, and by successive reflection from B and A respectively.

On moving the index arm C I towards S′, other objects, in addition to those seen directly through A, are brought into view, and if, when any particular object, in the direction C P, is brought into the field of view, the arm C I has been turned through an angle θ, then, by the principle of Art. **31**, the angle P C H′ is equal to 2θ. That is, the angle between an object seen in the direction C H′ and another object in the direction C P, is equal to twice the angle S C I.

Hence, in determining the angle between any two given distant objects, the instrument is first adjusted until one of the objects is seen directly through A, and also by reflections from B and A. The index I will then be at the zero of the scale on S S′. The arm C I is then moved until the other object, seen by reflections from B and A, appears to coincide with the first object still seen directly through the unsilvered part of A. The required angle is then obtained by doubling the angle S C I, which is given by the reading of the scale. Usually the scale is graduated on the principle of marking half-degrees as whole ones, so that the direct reading gives the required angle.

The sextant is used largely by navigators and surveyors. The sailor uses it to find the latitude. He first observes the altitude of the sun at noon, *i.e.* he measures the angle between the line joining him to the sun (when the sun is at its highest), and the line joining him to the point on the horizon immediately below the former line. Subtracting from this the dip of the horizon, and also subtracting (or adding) the sun's declination, *i.e.* the angle that the sun is north (or south) of the celestial equator, he gets the co-latitude. On subtracting this from 90° the latitude is obtained. The surveyor uses the sextant to find the angle subtended at the point of observation by two different points which may be in any plane.

120. The heliograph and heliostat. A heliograph is simply a plane mirror suitably mounted, so that by its means sunlight can be reflected from one station to another, say, several miles away. It is used for the transmission of messages; the mirror is covered and uncovered according to a given code, and the observer at the distant stations notes the duration and regularity of the flashes, and from this constructs the message.

The heliostat is simply a heliograph mirror in which by suitable means the reflected beam is sent in the same direction all day long. This is done by mounting the mirror on a frame drawn by clockwork, the mirror being moved so that its normal always bisects the angle between the direction of the sun and the direction in which the light is to be sent.

121. Paraboloidal Mirrors. In Art. 45 we saw that the aberration produced by spherical mirrors could be largely decreased by the use of stops.

A real *remedy* may be applied by the substitution of a *parabola* instead of a circle as the generating curve of the mirror. A parabola is a curve formed by a section of a cone parallel to its side, and a paraboloidal surface is generated by the revolution of this curve around its axis. A portion of a parabola of low angle is shown in Fig. 134. No matter how great the angle, parallel rays falling on a paraboloidal mirror parallel to the axis are converged accurately to a certain point called the focus, and, conversely, rays diverging from that point are accurately parallelised. Fig. 134 shows the form of mirror employed in lightships and railway signal lamps where it is necessary to parallelise rays as perfectly as possible.

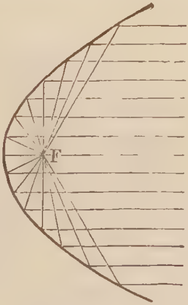

Fig. 134.

122. Cylindrical mirrors and lenses. These mirrors behave like plane mirrors for dimensions of objects parallel to their lengths and as concave or convex mirrors for dimensions perpendicular to their axes. They are used largely for shop-window illumination.

The lenses act similarly. In the laboratory they are often employed to produce images of illuminated slits; in the outside world they are used largely by spectacle-makers as a corrective to astigmatism (Art. 132).

123. Total reflection prisms. Let A B C (Fig. 135) represent the section of a prism with angles 90°, 45°, 45°. If an incident beam of parallel light, P Q, falls normally on the face A B, it suffers no refraction and very little loss by reflection, and meets the hypotenuse A C at an angle of 45°, which is greater than the critical angle (41°). Consequently it is *totally* reflected in the direction Q R, and reaching the face B C normally, it emerges without refraction and with very little loss by reflection from the surface B C. The beam is thus deviated through a right angle with very little loss of light.

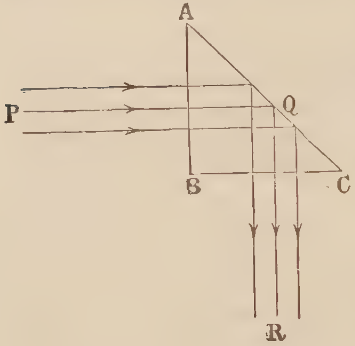

Fig. 135.

If a plane mirror of glass, silvered at the back, be employed to deviate a beam through a right angle, confusion is often caused by the succession of images which are formed (Art. 59). This error can be eliminated by silvering the front surface, but when we remember that even the most highly polished silver surface reflects regularly considerably less than the whole of the light incident upon it, and that there is difficulty in keeping a silver surface in good condition, we can easily see why such reflecting prisms are very frequently used in optical instruments. But such prisms can only be used to reflect light at an angle of incidence greater than the critical angle. Since the rays are intended to enter and leave normally, the prism should be isosceles with its angle B equal to the angle of deviation required. If used for other angles, there is more loss of light by reflection at B C or B A. The issuing beam is not identical with the original beam, but undergoes the same kind of reversal, right and left interchanging, as with reflection from a plane mirror.

Total reflection prisms are largely used in lighthouses. Right in front of the light is placed a large compound plano-convex lens, and around it arranged in rings are fixed a number of total reflection prisms of varying angle, such that all the light is sent out in a parallel beam. As important applications of total reflection prisms in optical instruments, see the Newtonian telescope, Art. **138**, Fig. 155, and the comparison spectroscope, Art. **143**, p. 255.

Wollaston's prism, sometimes called the *camera lucida*, is a totally reflecting prism with four angles, generally employed as an aid to sketching A section of the prism is shown in Fig. 136; the angle A B C is a right angle, A D C is 135°, and the other two angles each $67\frac{1}{2}°$. Light incident normally on B C, in the direction P Q, is totally reflected from the face D C to the face D A, whence it is totally reflected along R S normally to the face A B. To an eye

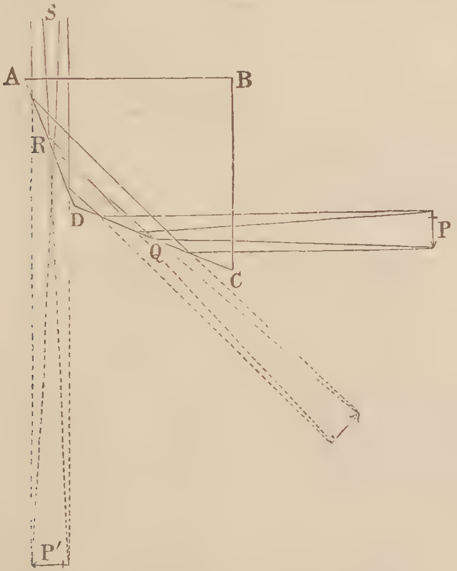

Fig. 136.

looking along S R, objects in the direction of Q P are seen in the direction S R P′, and the image thus seen may be traced on a sheet of paper placed at P′, vertically below S. The sheet of paper is seen past the edge A of the prism, while the image is seen by reflection from the face A D. It is important that the image should be in the plane of the paper, for then paper, pencil, and image are seen with the same focussing of the eye. For this reason a concave lens of short focal length is placed in front of the face B C when the object to be sketched is very distant. By adjusting the height of the prism in its stand (Fig. 137), the image can then be made to coincide with the plane of the paper. The two reflections at the faces C D and D A are necessary to give an erect image.

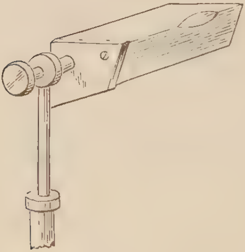

Fig. 137.

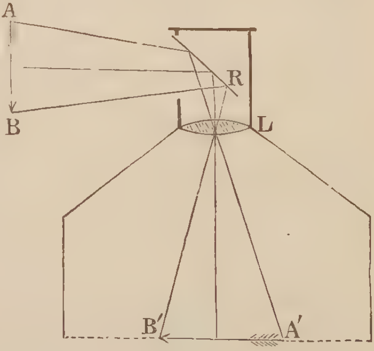

Fig. 138.

124. **The camera obscura.** The principle of this arrangement is indicated in Fig. 138. At the top of a small tent or wooden structure is a small cylindrical or cubical box, which contains a mirror, R, and a lens, L, arranged as shown in the figure.

The mirror is inclined at 45° to the horizontal, and reflects the rays coming from any external object A B on to the lens L, which forms an image A′ B′ on a white table or screen placed vertically below it. The room is perfectly dark and the inside carefully blackened, so that the image thus cast upon the screen or table may be clearly seen. The box containing the mirror and lens can be rotated, and thus images of all objects surrounding the tent are in turn cast upon the table. Instead of the mirror and lens it is better to employ a totally reflecting prism with the faces, which are turned towards A B and A′ B′ respectively, convex and concave. The curvature of the concave surface is less than that of the convex, and the arrangement thus acts as a convex lens and a mirror combined. Since the objects under observation are relatively far away, all the images on the table are in focus at the same time. Their sizes are the same as if a simple aperture were used instead of the lens and mirror.

125. Photographic camera. If we disregard all unessential details, this instrument may be described as a box (Fig. 139) with a convex lens * in front, and at the back a ground glass screen, which can be replaced by a slide carrying a sensitised plate. When this instrument with the lens uncovered is directed towards any object that is to be photographed, an inverted image of that object will be formed; and by sliding the lens or the back of the box in or out, the image may be accurately focussed on the ground glass screen.

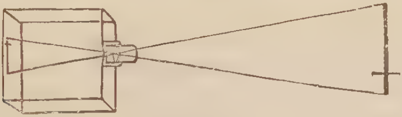

Fig. 139.

If a suitably prepared photographic plate † be then substituted for the screen, the action of the light on this plate is such that when subjected to proper chemical treatment a *negative* is obtained, from which the ordinary photographs can be printed on sensitised paper.

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* Usually an achromatic combination.

† The first photographs were taken by Daguerre in 1839.

The pinhole camera (Art. 8) is very useful for some kinds of photographic work, especially that of taking buildings, as it produces no distortion, whereas an ordinary lens gives images with strongly curved lines; and in fact many of the cheap cameras on the market are simply pinhole cameras. A disadvantage of such cameras is that the light can only enter through a small hole and thus long exposures are required, but by using extra rapid plates this difficulty can be surmounted.

126. Why optical instruments are blacked inside. Cameras, telescopes, microscopes, and other optical instruments are always painted dead black inside in order to prevent internal reflections. In the camera, for instance, when the light reaches the sensitised plate a considerable proportion is scattered in all directions, and falling on the sides, top, bottom, and front of the box, would, if these were light in colour, be in great part reflected back to the plate and fog it. In other instruments such internal reflections would similarly confuse the effect produced by the direct rays.

127. The optical lantern. In attending any course of experimental lectures this instrument will be seen in constant requisition for a great variety of purposes, and the student should have some idea of its construction and mode of action. It is now surely quite time to drop the old-fashioned name *magic* lantern, except for the toys used for exhibiting pictures only.

Essentially it is a light-tight box (Fig. 140) enclosing a powerful light or radiant, S. For scientific experiments the radiant should be a powerful one, but *it is much more important that it should be very small*, the smaller the better. The limelight and electric arc are satisfactory in both these respects. In front of the box is the *condenser*, L, usually a pair of plano-convex lenses about 4 ins. in diameter. The radiant is so mounted that it can be slid backwards and forwards in the optic axis of the condenser. When it is in the principal focus of the condenser a powerful parallel beam emerges. When slid further back the beam can be made to converge to any required point, and after the crossing of the rays to diverge from that point.

When used for projecting images of various pieces of apparatus or of pictures, an achromatic lens combination, of about 6 in. focus called the *objective*, O, is mounted in front of the condenser, and the radiant is placed so that the whole of the convergent beam issuing from the condenser may be taken up by the objective.

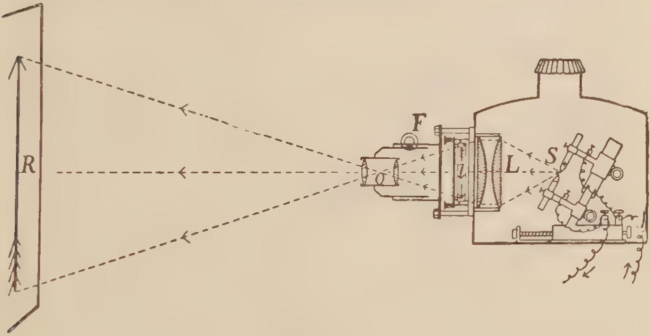

Fig. 140.

The lantern slide, *l*, which is a transparent photograph, or other suitably prepared representation of the object to be shown is placed between O and L, as close as possible to the latter, and the tube carrying the former is then screwed backwards or forwards until a clearly defined image is focussed on the screen R. The image is inverted and magnified in the ratio R O : O *l*. Since the intensity of illumination of the image varies inversely as the square of R O, the best magnification that can be obtained depends ultimately on the illuminating power of S. The image is inverted, hence to get an erect image the lantern slide is put in upside down. In the projection of a piece of apparatus on the screen inversion is often undesirable, and in such cases the beam is re-inverted by means of a total reflection prism.

The condenser is used simply to concentrate (or condense) the light on the slide. It plays no part in the focussing and hence need not be corrected for either kind of aberration.

128. The eye. The human eye is essentially an optical instrument, similar in principle to the photographic camera described in Art. 125. Fig. 141 shows, diagrammatically, a vertical section of the eye from front to back. Anteriorly we have the *cornea*, C, behind which is the anterior chamber of the eye, bounded behind by the *crystalline lens*, L, and the ciliary processes, *c c*, to which the lens is attached. This chamber is filled with a watery fluid called the *aqueous humour*, A; and in front of the crystalline lens lies the *iris*, I I, a circular curtain with a central aperture, *p*, called the *pupil*. The iris is seen in the eye as the coloured ring surrounding the pupil; it is a muscular structure made up of circular and radial fibres, so arranged that the size of the pupil can be increased or diminished as required. The *crystalline lens* L, is the lens of the eye; it consists of a soft transparent substance enclosed in a thin transparent membrane, and is held in position by the ciliary processes which are attached round its circumference. In structure it is somewhat complex; the posterior face is more convex than the anterior, and it is built up of a large number of concentric shells, increasing in density as they approach the centre, the outer shell having the same density as the surrounding medium. By this arrangement the optical action of the lens is more powerful than if it were composed of a homogeneous medium of the same density as the nucleus, and also the loss of light by reflection at the surfaces of the lens is diminished.

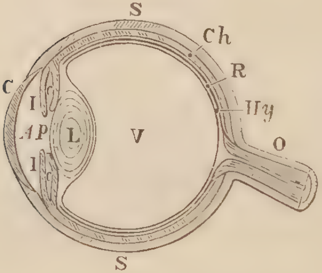

Fig. 141.

Behind the crystalline lens, in the posterior chamber, is the *vitreous humour*, V; this is a watery fluid very similar to the aqueous humour.

The walls of the eyeball are made up of three coats. The outer, S, called the *sclerotic*, is a tough white coat, giving consistency to the ball. The cornea, in the front, is fitted into this coat like a watch-glass into the case of the watch. The middle coat, *Ch*, the *choroid*, is a thin pigmented layer which divides in front into two layers; the anterior layer goes to form the iris, and the posterior constitutes the collar of ciliary processes carrying the crystalline lens. Adjacent to the ciliary processes is a muscular collar attached to the sclerotic coat, and inserted at the circumference of the crystalline lens. This collar is the *ciliary muscle*, and serves, by its action, to vary the convexity of the surfaces of the lens. The inner coat, R, is the *retina*, a delicate membrane which is practically a fine network expansion of the optic nerve, O. It covers the whole of the inner posterior surface of the eye as far as the ciliary collar, and is lined by a very delicate membrane, H*y*, called the *hyaloid membrane*. At the centre of the retina is the *yellow spot*, a small slightly raised yellowish spot, having a minute depression, called the *fovea centralis*, at its summit. This yellow spot, which is about $\frac{1}{20}$th of an inch in diameter, is the region of most distinct vision, and the fovea centralis is the most sensitive spot on the retina. About $\frac{1}{10}$th of an inch on the inner side of the yellow spot is the *blind spot*, the point at which the optic nerve enters the eye. This spot is not sensitive to light.

Considered optically, then, the eye consists of a double convex lens, the crystalline lens, protected in front by a circular diaphragm, the iris, and having a sensitive screen, the retina, on which the images of external objects are cast. The impressions conveyed to the brain by these images give rise to the sensation of sight.

In the case of the human eye the index of refraction of the cornea and the humours is 1·337, that of the lens is 1·437. The thickness of the cornea is 0·4 mm. and the radius of its front surface 7·8 mm. The thickness of the lens is 3·6 mm., the radii of its front and back surfaces are 10·0 and 6·0 mm. respectively, and its front surface is 3·2 mm. behind the cornea.

129. Vision. The condition of distinct vision of any object is that a clearly defined image of it is formed on the retina. Fig. 142 represents such a case. When light enters the eye refraction occurs at the surface of the cornea and of the crystalline lens.

The centres of these surfaces lie on a straight line called the optic axis, which meets the retina between the yellow and blind spots. Since no change of refractive index occurs from the cornea to the aqueous humour, the aqueous humour may be regarded as extending to the anterior surface of the cornea. If the object is large the image is distorted, due partly to obliquity of the extreme rays and partly to spherical aberration. The distortion is, however, greatly corrected by the heterogeneity of the crystalline lens and by the spherical shape of the eyeball.

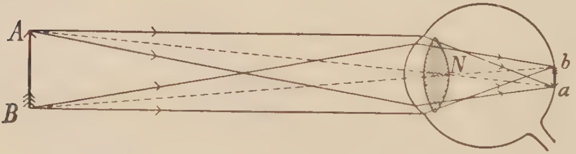

Fig. 142.

In all cases in which real objects are distinctly seen real inverted images are produced on the retina; that we see objects erect is due to the interpretation which the brain puts upon the stimuli it receives.

Under ordinary conditions it is evident that, with an eye as above described, only objects at a certain definite distance from the eye can be seen distinctly; for, the distance between the image and the lens being fixed, the distance between the object and the lens must also be fixed. We know, however, from experience that objects can be seen distinctly by the normal eye at all distances greater than a certain minimum limit known as *the distance of nearest distinct vision.* This is due to the power of *accommodation* possessed by the eye; the ciliary muscle, we have seen, is able to alter the curvature of the surfaces of the lens, making the front surface much more convex, and bringing the lens as a whole nearer to the cornea; thus the focal length is *accommodated* to the distance of the object on which the eye is focussed. For a normal or *emmetropic* eye the limits of distinct vision are from a point distant about 10 inches from the eye to infinity. When the eye is at rest, it is supposed to be adjusted for parallel light—that is, for distinct vision of very distant objects.

The eyes of various individuals vary much in their accommodative power. Young children can see distinctly objects placed two or three inches in front of their eyes, ordinary adults can see objects as near as ten inches; but as the age of a person advances the power of accommodation of the eye decreases, probably because of a loss of elasticity in the outer layers of the crystalline lens. This defect of vision is called *presbyopia*, and it causes the nearest point of distinct vision to gradually recede from the eye. Thus in order to read a book an old man is often compelled to hold it at arm's length.

130. Magnifying power. The absolute size of an object is, of course, a constant quantity, but the *apparent size*, which is proportional to the magnitude of the retinal image, depends upon the distance of the object from the eye. A measure of this apparent size is given by the angle which the object subtends at the eye. This angle is called the *visual angle*. Thus if A B, Fig. 143, represent the object and E the eye, the apparent size is measured by the visual angle A E B, and this angle evidently decreases as the distance of A B from E increases. When the angle is small the visual angle is approximately measured by the

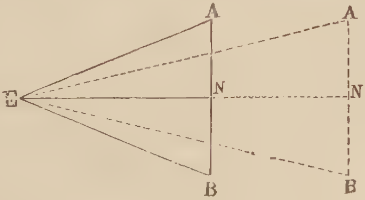

Fig. 143.

ratio $\frac{AB}{EN}$, or may be taken as proportional to the tangent of the angle A E B—that is,

$$\text{tangent (visual angle)} = \frac{\text{Length of object}}{\text{Distance of object from eye}}.$$

When we look at a building a mile away, the visual angle under which we see it is very small, consequently its image is very minute, and we can perceive nothing but its general form. At 100 yards it subtends a much larger angle, and its image may perhaps occupy almost the whole of the retina, and we are able to perceive doors, windows, and smaller features. At 20 yards only a portion can occupy the retina at one time, and that portion subtends a much greater angle than before. It may perhaps contain a printed bill, and the larger type may be easily read. At 1 yard the retina may be wholly occupied by the image of the bill, and all but the smallest type may be read. At 10 inches one of the smallest letters subtends such an angle that its form is plainly perceptible. At 4 inches the retinal image of that letter is larger still, *but its form is no more distinct*—it is less so, because the rays proceeding from the letter now diverge so widely from it that they cannot be focussed on the retina. Therefore nothing is gained in the way of distinct vision by any closer approach than 10 inches.

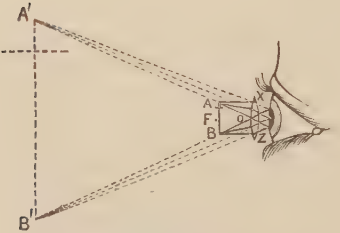

Fig. 144.

We have adopted this line of discussion in order to give the student a clear idea of the mode of action of the **Magnifying Glass.** A magnifying glass is simply a convex lens of short focal length employed to obtain magnified *virtual* images of small objects. The lens is placed at a distance less (usually *very little* less) than its focal length from the object to be viewed, A B (Fig. 144), and, as explained in Art. 78, a virtual magnified image is formed at A′ B′, which, when the position of the lens is properly adjusted, can be clearly seen by an eye at E. But it will be noticed that the enlargement of the image is counteracted by its increased distance, and the visual angle A′ O B′ under which it is seen, and therefore the size of the retinal image, is only the same as that of the object itself. Where, then, does the magnification come in? If the object were brought as close (perhaps 1 inch) to the unaided eye as it is to the lens, its retinal image, though equally large, would be indistinct because of the inability of the eye to focus such highly divergent pencils. What the lens does is to reduce this great divergence X A O, Z B O, to the much smaller divergence X A′ O, Z B′ O, and so allow the rays to be focussed on the retina, while preserving the great visual angle.

131. Magnifying-power of lens. Since the size of the retinal image is inversely proportional to the distance of the object, and this distance is determined by the focal length of the lens, than which it is very little less, the magnifying-power may be taken as the quotient of the near point of distinct unaided vision (usually taken as 10 inches) divided by the focal length of the lens. Therefore the magnifying power of a lens of 2-inch focus is $\frac{10}{2} = 5$, and of a lens of $\frac{1}{2}$ inch focus is $\frac{10}{.5} = 20$. This, which is what is usually understood as magnifying-power, refers to linear dimensions. Of course the superficial magnification is the square of the linear.

The above is only approximate, for the object is not placed quite at the principal focus. Let the distances of object and image from lens be denoted by u, D, respectively, where D is the least distance of distinct vision, and let $f =$ focal length of the lens. Then

$$\frac{1}{\mathrm{D}} - \frac{1}{u} = \frac{1}{f}. \quad \therefore u = \frac{-f\mathrm{D}}{f - \mathrm{D}}.$$

But the magnification, in general equal to v/u, is in our case equal to D/u,

$$i.e.\ \frac{\text{Image}}{\text{Object}} = \frac{D(f-D)}{fD} = \frac{f-D}{f} = 1 - \frac{D}{f}.$$

Or, since we are dealing with a convex lens, we may write magnifying-power $= 1 + \frac{D}{f_1}$, where f_1 is the *numerical* value of the focal length, and D is the least distance of distinct vision for the eye considered. Thus if f_1 is equal to 2 ins., the magnifying-power is 6, and not 5, as obtained by the approximate method above.

Example. The power of a lens (see Art. 82) is 40, find the magnifying power.

The least distance of distinct vision is about 10 ins., *i.e.* 25 cms. or a quarter of a metre. Let P denote the power; then, working in metres, we have the magnifying-power

$$= 1 - \frac{D\text{ (in cms.)}}{f\text{ (in cms.)}} = 1 - \frac{D\text{ (in metres)}}{f\text{ (in metres)}} = 1 + PD = 1 + \frac{P}{4}.$$

For the given lens P = 40, ∴ magnifying power = 11.

When an optical instrument is said to *magnify* an object, it is meant that the visual angle of the object as seen through the instrument is greater than its visual angle when seen directly by the naked eye, and the *magnifying-power* of the instrument is measured by the ratio of the visual angle as seen through the instrument to the visual angle when seen directly. This definition is evidently not sufficiently precise, for the visual angle of an object, seen directly, depends upon its distance from the eye; it is therefore necessary to specify this distance. In the case of a telescope, where the object viewed is distant, the magnifying-power is defined as the ratio of visual angle of the image seen in the telescope to the visual angle of the object seen directly *at its actual distance from the eye.* In the case of the microscope, however, the object viewed is near at hand, and it is assumed that when seen directly it is placed at the distance of nearest distinct vision, where its visual angle is greatest. Hence the magnifying-power of a microscope is defined as the ratio of the visual angle of the image seen in the microscope to the visual angle of the object seen directly *at the least distance of distinct vision.*

132. Defects of vision; Spectacles. The most common defects are known as (1) **Myopia** or **Short-sightedness**, (2) **Hypermetropia** or **Long-sightedness**, (3) **Presbyopia**, (4) **Astigmatism**.

A normal or emmetropic eye brings parallel light to a focus on the retina. By means of its power of accommodation the eye can also focus light from nearer points. Let N be the nearest point of distinct vision (Fig. 145), then images of all points on the line from N to + ∞ can be focussed on the retina by the unaided eye.

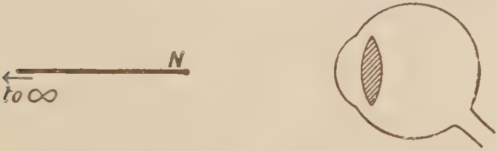

Fig. 145.

1. In **myopic** eyes either the axis of the eye is too long or the crystalline lens is too convergent. Light from a distant object is brought to a focus in front of the retina (Fig. 146), and thus the object is either not seen at all or seen very indistinctly. As the object approaches the eye the image travels backwards from the focus of the lens, and when the object reaches a point P, a certain distance away, the image falls exactly on the retina. This point P is the point of farthest distinct vision, which, if the eye were normal, would be at infinity. If the accommodating mechanism be perfect the eye will, up to a certain limit, be able to adjust itself so as to give distinct vision for objects

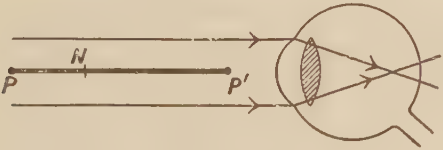

Fig. 146.

nearer the eye than this point, and even nearer than N, the near point for normal eyes. Let P′ be the nearest point of distinct vision for a myopic eye. For the unaided eye only those points can be distinctly seen which lie between P and P′. Rays from a distant point are brought to a focus in front of the retina, even when the muscles of the eye make it as little convex as possible. This defect can be remedied by the use of spectacles. To determine the nature of the lenses required, we must notice that the necessary condition for remedying the defect is that rays diverging from a point on the *normal* range of vision, *i.e.* ∞ N, should, after refraction through the lens, apparently diverge from a point within the range of the short-sighted eye, *i.e.* P P′. Suppose this latter range to be from 3 to 8 inches from the eye. Now we cannot make this coincide with the normal range at *both* ends; we must therefore decide either for coincidence of the nearest or of the farthest points of distinct vision. It is usual to choose the latter, because it corresponds to the quiescent state of the eye; hence, in this case we require a lens such that rays coming from infinity—that is, parallel rays—will, after refraction through it, diverge from a point F, nearly 8 inches from the eye (Fig. 147). The required lens is therefore a *concave* lens of 8 inches focal length; and, if x denote the nearest point of distinct vision with this lens, we have—

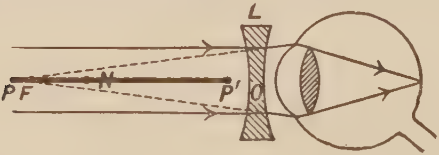

Fig. 147.

$$\frac{1}{3} - \frac{1}{x} = \frac{1}{8} \quad \therefore x = 4{\cdot}8 \text{ ins.}$$[*]

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[*] Determined by the condition that rays diverging from a point x inches in front of the lens must, after refraction, appear to diverge from a point 3 inches in front of the eye.

That is, the range of vision is now from 4·8 inches to infinity, instead of from 3 inches to 8 inches in front of the eye.

A practical consequence of the use of a concave lens is that the retinal images are diminished, and thus objects appear smaller than they do to normal eyes.

2. In the case of **hypermetropia** or long-sightedness the axis of the eye is too short or the lens not sufficiently convergent. When unaccommodated the only light which can be focussed on the retina is that which is converging to a point P (Fig. 148) *behind* the eye.

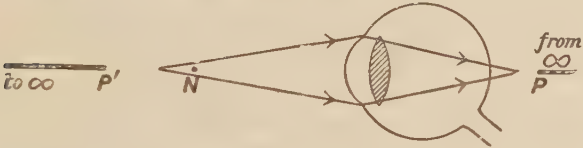

Fig. 148.

By means of accommodation, rays can be focussed which are converging to points on the line from P to $-\infty$, and rays which are diverging from points between $+\infty$ and P′, the nearest point of distinct vision. P′ is at a greater distance from the eye than N, the nearest point of distinct vision for the normal eye.

Rays diverging from points nearer than P′ are focussed behind the retina. The power of the eye lens (to converge the rays) is too little, and so a convex lens must be requisitioned to help it. Suppose, for example, P′ is 30 inches in front of the eye and P 10 inches behind; then, to render the farthest points of distinct vision for the unaided normal eye and the aided hypermetropic eye coincident, we must employ a lens such that rays coming from infinity will, after refraction through it, converge to the point P 10 inches behind the lens—that is, a *convex* lens of 10 inches focal length must be used, and the distance of the nearest point of distinct vision (x) is given by the relation—

$$\frac{1}{30} - \frac{1}{x} = -\frac{1}{10};$$

$$\therefore\ x = 7\tfrac{1}{2} \text{ ins.}$$

That is, the range of vision is now from 7½ inches to infinity, instead of from 30 inches in front of the eye up to and through $+\infty$, and from $-\infty$ back to 10 inches behind the eye.

The action of the lens is shown in Fig. 149. The effect of L is to push P to $+\infty$ and to bring P′ nearer to N. Another effect of the convex lens is to make objects appear larger than they appear to the normal eye.

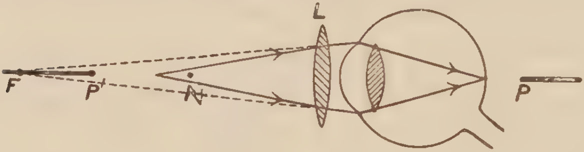

Fig. 149.

3. In the case of **presbyopic** eyes (usually found in old people, Art. 129) distant objects can be distinctly seen, but light from near objects cannot be focussed on the retina—that is, the nearest point of distinct vision has receded. To remedy this defect by means of spectacles, the focal length of the lenses must be adapted to the purpose for which the spectacles are required. For reading purposes the lenses must be chosen so as to cause rays diverging from the point of normal nearest distinct vision to appear to diverge, after refraction through them, from the nearest point of distinct vision of the defective eye. For example, suppose the nearest point at which a person can see distinctly is 30 inches; then, if f denote the focal length of the lenses required, we have—

$$\frac{1}{30} - \frac{1}{10} = \frac{1}{f};$$

$$\therefore f = -15 \text{ ins.}$$

That is, convex lenses of 15 inches focal length are required.

When the accommodating mechanism is not perfect, there may be practically only one point of distinct vision, and the defect can be remedied only for particular cases. For example, suppose a person is able to see distinctly only at a point distant about 4 inches from the eye, then, if he requires spectacles to enable him to see distant objects distinctly, the focal length of the concave lenses to be used must be about 4 inches. If, however, he requires spectacles for reading purposes, and he wishes to hold his book in the same position as a normal-sighted person holds his, then the focal length of the lenses is determined from the fact that light coming from objects at the normal distance of distinct vision, 10 inches, should, after refraction, appear to come from a point 4 inches from the eye. Therefore—

$$\frac{1}{4} - \frac{1}{10} = \frac{1}{f},$$

or—

$$f = 6\tfrac{2}{3} \text{ ins.}$$

That is, concave lenses of $6\frac{2}{3}$ inches focal length are required.

4. In the case of **astigmatic** eyes—due mainly to non-sphericity of the cornea—a vertical section being usually more curved than a horizontal section—lines inclined in one direction can be seen much more plainly than lines in a direction perpendicular to this. There are very few eyes that do not suffer from this defect, horizontal lines being usually brought to a focus in front of the focus of vertical lines. A test may be made by drawing four or five parallel lines close together on a sheet of paper, which is then placed facing the patient about 4 or 5 yards away and slowly rotated. The patient with one eye open watches the lines, and in general it will be found that for quite a large range of rotation they appear very indistinct. To remedy this defect a cylindrical lens (Art. 122) is required, its position being arranged so that the refraction which it produces is in the same way as the weaker refraction of the cornea. If the eye is myopic or hypermetropic as well as astigmatic, a lens cylindrical on one side and spherical (concave or convex) on the other side will be required.

In all cases of defect of vision the magnitude of the defect may not be the same for both eyes, so that lenses of different focal length may be required to accurately correct the vision.

133. Miscellaneous experiments and observations with the eye.

(1) To show that the eye is over-corrected for spherical aberration. Bring a printed page so close to the eye that the print is indistinct. Now interpose a sheet of paper with a pinhole in it between page and eye and just in front of the latter. The print seen through the hole is quite distinct. This shows that rays going through the centre of the lens are converged more than those going through the peripheral portions. The opposite occurs with ordinary lenses. (Art. 83.)

(2) To show that the eye is not achromatic for the extreme rays though very nearly so for intermediate rays. Looking at a window frame with a bright background and holding a finger close in front of the eye, gradually move it across the field of view. As the advancing finger approaches a bar of the window-frame the near edge of that bar will bear a blue fringe, and the far edge a red fringe. In trying to understand this remember that the image on the retina is inverted.

Another method of showing the existence of chromatic aberration in the eye is to bring a well-illuminated printed page close up to the eye and stare at it. Blue and yellow fringes will then be seen to all the letters.

(3) Images of real objects are inverted. Experience, however, tells us that the objects are the right way up. If therefore an erect shadow could be thrown on the retina, it should appear inverted. To verify this make a pinhole in a piece of paper and holding it about an inch or so in front of the eye view through it a brightly illuminated surface such as a white lamp globe. Take now a pin and holding it head upwards introduce it between the eye and the pinhole. As shown in Fig. 150 the pin appears inverted.

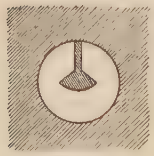

Fig. 150.

(4) To prove the existence of the blind spot. Close the right eye and keeping *the left eye fixed on B* (Fig. 151) move the book to and from the eye. When the book is about 12 inches from the face A will be invisible, but will come into view again for greater or lesser distances. Repeat with only the right eye open and fixed on A.

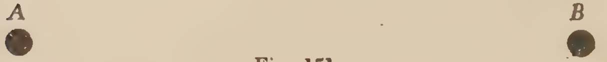

Fig. 151.

(5) The retina continues to feel the effects of the light after the exciting agent has been removed. This phenomenon is called the **persistence of impressions**, the impressions lasting about one-tenth of a second. Thus the glowing end of a match stick yields a bright

circle when the match is swung around, and not a bright point changing its position. Again the colours of a rapidly rotating colour disc (Art. 106) blend into one; and an alternating electric current gives a steady light, the fluctuations of the light being too rapid to be noticed, except it be used to illuminate a rapidly moving object.

(6) Look at a bright object, such as a bright lamp globe, and then close the eyes, and, in addition, cover them. An image of the globe will appear; this is the *positive after-image.* After a time it disappears, and may be followed by a very dark image of the globe on the fairly dark background; this is the *negative after-image.* The positive image is due to continued nerve irritation, similar to the persistence of impressions, whilst the negative image is due to the fatigue of those nerves upon which the bright image fell. This prevents them being excited to the same extent, as the unacted-on nerves are, by the dull light penetrating the eyelids; hence the dark image. Repeat the experiment with a strongly illuminated coloured form, such as a church window. The positive after-image in this case is similar in colour to the window; it then fades away, and the negative after-image appears in the complementary colours.

Repeat by looking steadily at a bright red spot, and then shifting the eyes to a dull grey surface. A greenish-yellow spot will be seen on it. Other contrast results may be obtained by placing a small white circle of paper upon larger pieces of coloured papers. In each case the white seems to be illuminated by the colour complementary to that of the background. Thus on yellow it appears blue, and on green it appears of a ruddy hue. Again, if a hole be made in a piece of bluish green glass, and the glass be then held between the sun and a white screen, the shadow of the hole will look dark pink.

(7) Close the eyes and press with the fingers into one of the hollows on the upper side of the eyes next the nose. A circle of light will appear at the opposite side of the eyeball. This shows that the nerves may be stimulated by mere pressure from the outside.

(8) If we have two bodies of the same size, one bright and the other dark, the former will look the bigger of the two. The portion of the retina which is being excited extends beyond the geometrical image of the body seen. The most striking example of irradiation occurs when the moon is in her first quarter, and is called "the old moon in the young one's arms." The illuminated crescent (Fig. 152) then appears to be a part of a much larger circle than that of the faintly illuminated new moon.

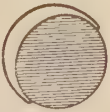
Fig. 152.

(9) A peculiar wavelike motion is often observed in a row of close-set railings when one is walking near by and looking through them at a farther row. At certain places bars in the two rows are behind each other, and thus the maximum amount of light is able to penetrate them. At other places bars in the farther row are behind spaces in the front row, and the illumination is smaller, varying to a minimum

when the bars are behind the centres of the spaces. The same phenomenon is also very noticeable when a meat safe of perforated zinc is under observation, and when a piece of wire gauze is lying upon another. The appearance is very much like that of "watered silk."

(10) The **distance of objects** is judged partly by the amount of accommodation it is necessary to impress upon our eyes in order to see them distinctly, and partly by the amount of convergence between the optic axes of the two eyes. When the distance, however, exceeds a certain amount the accommodation is constant, and the optic axes are sensibly parallel. Hence other methods of judgment must be used, the usual being that of comparing the size of a known object at the far distance with the apparent size it would have when close to the observer. In judging distances, therefore, practice counts a lot. On a very clear day distant objects appear nearer than they really are, and hence we judge their magnitude smaller. In a fog vision is indistinct, and, as we associate indistinction with distance, we unconsciously estimate the object to be some distance away, and therefore larger than it really is.

Again, the sun and moon usually look larger when low down than when high up in the sky. This false impression is not in accordance with measurements of the angular diameter made by a micrometer. When near the horizon the eye is apt to estimate the size and distance of the sun and moon by comparing them with the neighbouring terrestrial objects (trees, hills, etc.). When the sun and moon are at a considerable altitude no such comparison is possible, and a different estimate of their size is instinctively formed.

EXERCISES IX.

1. A person's range of distinct vision is from 4 inches to 8 inches from the eye; find the focal length of the lenses he should wear, and the range of his vision with those lenses.

2. Explain, by help of a diagram, the effect of a convex lens held close to the eye and employed as a simple microscope.

Prove an approximate formula for the magnifying power of the lens, its principal focal length and the distance of distinct vision by the naked eye being given.

3. A person whose nearest distance of distinct vision is 18 inches uses a reading lens of 6 inches focal length; what magnification does he obtain?

4. A person can see objects distinctly only at a distance of about 4 inches from the eye; calculate the focal length of lenses he should use for reading, walking, and for viewing distant objects. Assume that 10 inches is the normal distance of nearest distinct vision, and that in walking the average distance at which he requires to see clearly is 15 feet.

5. An aged person sees distinctly from infinity up to about 20 inches from the eye. What spectacles should be worn to remedy this defect?

6. A lantern slide is 3¼ inches square and an enlarged image is to be formed on a screen 20 feet distant from the lens by the aid of a lens of 6 inches focal length. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

7. Explain why it is possible to take a photograph with a pinhole pierced in an opaque screen in place of a lens, and why it is not possible to do so when the hole is large.

8. A pinhole camera is made in the form of a cube (edge 1 foot) with a hole in the centre of one side. It is placed opposite a building 60 feet high at a distance of 100 yards. Find the size of the image.

9. A total reflection prism is employed to deviate a ray through 60°. What is the shape of its section?

10. A person is under water. Do objects in the water look at the same distance from him as they would in air? Why are near objects indistinct?

11. A large flat air-tight pinhole camera is placed, with the pinhole pointing upwards, on the bottom of a pool of water surrounded by houses. A photographic plate is exposed in the camera. Explain what will be the general appearance of the picture obtained on development.

CHAPTER XII.

MORE COMPLEX OPTICAL INSTRUMENTS.

134. In this chapter we shall consider the more complex optical instruments—telescopes, microscopes, spectroscopes, etc.

135. The telescope. Telescopes are employed for the purpose of obtaining distinct vision of distant objects, especially stars and other celestial bodies. They are of two kinds—*refracting* and *reflecting*, but the same general principle underlies them all.* A real image of the object is formed by a convex lens (*object glass*) or concave mirror (*speculum*), and is examined by a magnifying glass (*eye-piece*). There are several different *forms* of telescopes, the details of construction in each case being adapted to the purpose for which that particular form is intended.

136. The refracting astronomical telescope. This instrument was invented by Kepler, the astronomer, in 1611, but was first used by Huyghens in 1655. In its simplest form it consists of a convex lens, O, fixed at one end of a brass tube (Fig. 153), and another and smaller convex lens, O′, fitted in a tube which slides inside the former. The lens O, which first receives the rays from the distant object, is called the *object glass*, and may be a simple convex lens; but in the best instruments it is a compound achromatic lens (Art. 99), having its focus at F. The lens O′, at the other end of the tube, may also be a single lens, but generally consists of a system of two lenses, called an *eye-piece*, the focal length of the system, O′ F′, being considerably smaller than that of the object glass. Considering the simple form shown in Fig. 153, the optical action of the instrument may be explained as follows. Let A, A, A, denote rays coming from a point A on a distant* object A B.† These rays, after refraction through the object glass, O, are brought to a focus at *a*. Similarly the rays B, B, B, coming from a point B on the object, are brought to a focus at *b*, and a real inverted image of the distant object is obtained at *ab*. By moving the eye-piece tube the position of the lens O′ can be adjusted so that the image *ab* falls just within its focal length—that is, the distance of *ab* from O′ should be slightly less than the focal length of O′. With this adjustment an eye looking through O′ sees a virtual magnified image of *ab* at A′ B′ (Art. 78, 1, 2). If the instrument is focussed so that A′ B′ is seen at an infinite distance from the eye, then *ab* is at the focus of O′. Also, if the object A B is distant, then *ab* is practically at the focus of O. Hence, under these conditions, the image *ab* is at a point which is the common focus of O and O′, and the length of the telescope is equal to the sum of the focal lengths of the object glass and eye-piece. For more distinct vision, however, the eye-piece is often focussed so that A′ B′ is seen at the nearest distance of dis-

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* The opera or field glass is not included in this general statement. It is also possible to construct a telescope entirely of prisms.

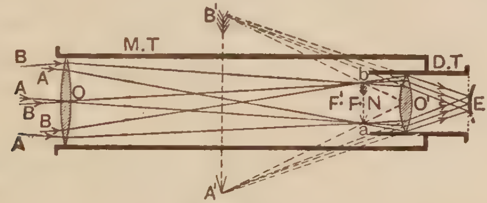

Fig. 153.

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* If the object is very distant, then the rays A, A, A, are practically parallel.

tinct vision, the distance of the image *ab* from O′ being then less than the focal length of the eye-piece, and the length of the telescope thus slightly less than the sum of the focal lengths of object glass and eye-piece. If a near object is being viewed, the distance of *ab* from O is greater than the focal length of the object glass; hence, in this case the length of the telescope is greater than the sum of the focal lengths of the object glass and eye-piece. It will be noticed that the image *ab* is inverted, and that, since A′ B′ is an erect image of *ab*, the image seen on looking through the instrument is inverted; this is immaterial in astronomical observations, but for terrestrial purposes it is necessary to have an erect image. In order, therefore, to adapt an astronomical telescope to ordinary use, it must be fitted with an *erecting eye-piece.*

137. Determination of the magnifying-power of a telescope. In order to determine the magnifying-power of a telescope it is only necessary to obtain the ratio of the visual angle (Art. 130) of the image to that of the object, the latter being seen at its usual distance from the eye. If this distance is great, compared with the length of the telescope, then, in Fig. 153, A O B is practically the angle which the object A B subtends at the eye. Similarly A′ O′ B′ is the angle which the image A′ B′ subtends, and the magnifying-power is given by the ratio $\frac{A'OB'}{AOB}$. But the angle A O B is equal to *a* O *b*, and A′ O′ B′ is identical with *a* O′ *b* (cf. Figs. 100, 144); therefore we have $\frac{A'O'B'}{AOB} = \frac{aO'b}{aOb}$; and, the angles involved being small, the magnifying-power is therefore approximately equal to $\frac{ab}{NO'} \Big/ \frac{ab}{NO} = \frac{NO}{NO'}$ where *N* is the middle point of *AB*. Now, if the object is very distant, the image *a b* is formed close to the principal focus of the object glass; and, if the position of O′ is adjusted so that *a b* is very near to its principal focus, then F and F′ coincide with N, which then becomes the *common* focus of O and O′, and *the magnifying-power is given by the ratio of the focal length of the object glass to the focal length of the eye-piece.* That is, if *m* denote the measure of the magnifying-power, F_1 the numerical value of the focal length of the object glass, and f_1 the numerical value of the focal length of the eye-piece, then—

$$m = \frac{F_1}{f_1}.\text{*}$$

This, it must be remembered, is true only when the object is very distant, and when the eye-piece is placed so that the image $a\ b$ is at its focus, and the virtual image A′ B′ is therefore at infinity. A slight increase in magnifying-power is obtained by focussing so that A′ B′ is seen at the nearest distance of distinct vision.

With this adjustment the magnifying-power is $\frac{F}{f}\left(1 - \frac{f}{D}\right)$ where D is the shortest distance of distinct vision. For, the image A′ B′ being now at a distance D from O′, the distance of $a\ b$ from O′ is given by—

$$\frac{1}{D} - \frac{1}{u} = \frac{1}{f}$$

where u denotes the required distance. That is—

$$\frac{1}{u} = \frac{1}{D} - \frac{1}{f} = \frac{f - D}{f D}.$$

$$\therefore u = \frac{f D}{f - D},$$

and the magnifying-power, being now determined by the ratio $\frac{F}{u}$ is equal to $\frac{F\ (f - D)}{f\ D}$; or, substituting numerical values and neglecting sign,

$$m = \frac{F_1\ (D + f_1)}{f_1\ D} = \frac{F_1}{f_1}\left(1 + \frac{f_1}{D}\right).$$

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* The sign convention is here neglected. Strictly m is *negative* because the image is *inverted*; this is, seen from the ratio $\frac{N\ O}{N\ O'}$, for $N\ O = F$, and $N\ O' = -f$; that is, $m = -\frac{F}{f}$.

Exp. 57. To find the magnifying-power of a telescope.

(1) Determine F and *f* by ordinary methods and substitute in the above equations.

(2) Focus the telescope on a distant object, such as a slate roof or a brick wall. Arrange that the image is formed in the plane of the object and thus all parallax is avoided. With one eye at the telescope and the other unassisted, then note how many slates or bricks as seen by the unaided eye occupy the same length as one slate or brick seen through the telescope. This number is the magnifying-power.

(3) Focus the telescope for infinity and then point it towards a bright cloud or a strongly illuminated surface. Observe the bright circle on the eye-piece. Measure its diameter $a\ b$ (Fig. 154) by means of a fine scale, and also the diameter A B of the object glass. Then clearly,

$$\frac{A\,B}{a\,b} = \frac{F_1}{f_1} = m.$$

Nearly all telescopes contain stops (Arts. **45**, **83**) whose function is to cut off the marginal rays proceeding from the object glass. In Fig. 154 the presence of S virtually diminishes the diameter of the object glass from A B to A′ B′, with a proportional decrease from $a\ b$ to $a'\ b'$ of the diameter of the circle of light on the eye-piece. To eliminate this error place a wide adjustable slit in front of the object glass and narrow it down until its shadow begins to encroach on the circle $a'\ b'$. Then carefully adjust it so as just not to encroach. The ratio $\frac{A'\,B'}{a'\,b'}$ is the magnifying power.

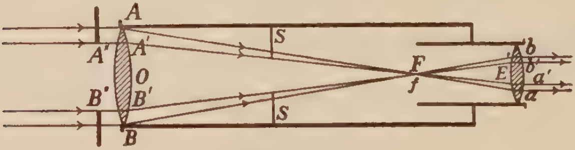

Fig. 154.

138. The reflecting telescope. This assumes many forms. The earliest described was invented by Gregory in 1663. In the most common, or *Newtonian* form (Fig. 155), invented in 1668, the end of the tube turned towards the object is open, and at the other end is a concave mirror or *speculum*, S, of glass silvered on the front surface.

This, if it were allowed to, would form at its principal focus a real image *a b* of the object to which it is directed. But before reaching the focus, the rays fall on a total reflection prism (Art. 123), which diverts the image to *a′ b′*, where it is examined by an eye-piece E inserted in the side of the tube.

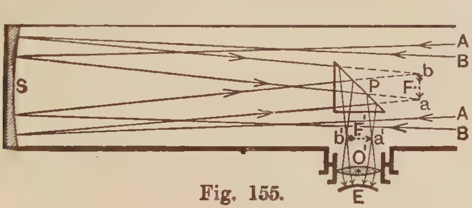

Fig. 155.

A great advantage of a reflecting telescope over a refracting telescope is that no chromatic effects are introduced at reflection; also, if required for observation on celestial bodies, aberration can be completely eliminated by using a paraboloidal mirror (Art. 121).

139. Why astronomical telescopes are made large. When an image is highly magnified, it is, of course, proportionately reduced in brilliancy. Many terrestrial, and most celestial, objects are themselves comparatively faint. It is therefore desirable that the telescope should be able to grasp as much light as possible from the object. This is the reason for the employment of object glasses or specula of large diameter or *aperture*, for the light-grasping power is proportional to the square of the diameter. By so doing many stars which are invisible to the naked eye are revealed. A long focus, and therefore a long tube, is used to give great magnifying-power, and also to decrease aberration by keeping the curvature of the reflecting or refracting surfaces low.

As a rule, astronomical telescopes are provided with several eye-pieces of different focal lengths. This allows the observer a range in magnifying-power which is most convenient, as some objects will stand a greater magnification than others.

Achromatic refracting telescopes give better results than reflecting telescopes of the same size, but owing to the immense care required in the manufacture of good lenses over two feet in diameter, most very large telescopes are reflectors. Lord Rosse's large reflector erected at Parsonstown, Ireland, in 1845, has a mirror of 6 feet in diameter and focal length of 53 feet. The largest refracting telescopes are very costly and hence are to be found chiefly in America. The Lick and Yerkes instruments have apertures of 36 and 40 inches respectively, and focal lengths of about 60 feet.

140. Galileo's telescope. This form of telescope was invented by Galileo in 1609. It consists, like the astronomical telescope, of an object glass and an eye-piece. The object glass is exactly similar to that of the astronomical telescope, but the eye-piece is different; it is simply a double-concave achromatic lens placed between the object glass and its principal focus, and at a distance from the latter equal to or slightly greater than its own focal length. The optical action of this arrangement will be understood on reference to Fig. 156.

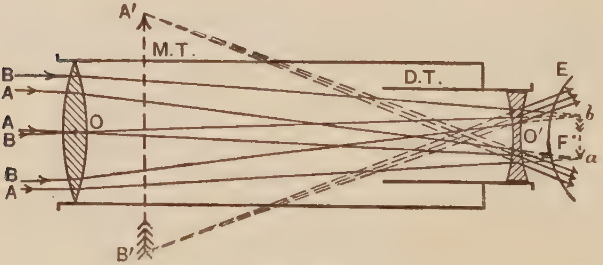

Fig. 156.

O represents the object glass, having its focus at F, and O′ the eye-piece, placed so that the distance O′ F is equal to or slightly greater than its focal length. We shall suppose that O′ F is slightly greater than the focal length of O′, so that F is nearer to O′ than the principal focus of O′. The rays A, A, A, and B, B, B, coming from points A and B of a distant object, would, if uninterrupted, form an image *a b* at a point just beyond F, but falling on O′, these rays are refracted so as to apparently diverge from the points A′, B′. A virtual erect image of A B is thus seen at A′ B′. The position of this image depends upon the position of O′ relative to *a b*; if the distance from O′ to *a b* is equal to the focal length of O′, then A′ B′ is at infinity; but, if this distance is greater than the focal length, then A′ B′ is nearer, and may by adjusting the position of O′ be brought to the nearest point of distinct vision, as in the figure. For the purpose of easily effecting this adjustment the eye-piece is mounted in a draw tube D T, which slides in the main tube M T of the telescope.

The magnifying-power is given, as in the case of the astronomical telescope, by the ratio $\frac{a\,\mathrm{O}'b}{a\,\mathrm{O}\,b}$; that is, if the object viewed is very distant and the image A′ B′ is at infinity—

$$m = \frac{a\,b}{\mathrm{O}'\,\mathrm{F}} \bigg/ \frac{a\,b}{\mathrm{O\,F}} = \frac{\mathrm{O\,F}}{\mathrm{O}'\mathrm{F}} = \frac{\mathrm{F}}{f}$$

where F and *f* are respectively the focal lengths of the object glass and eye-piece.

The chief advantages of this form of telescope over the astronomical are, that the construction of the instrument is simplified, its length is reduced, and an erect image is obtained directly without the aid of a special eye-piece. On the other hand, the disadvantages are numerous; the errors of aberration are imperfectly remedied, cross wires cannot be used, and the magnifying-power and field of view are very limited. This last disadvantage arises from the fact that the eye-ring (*i.e.* the region where the dotted rays from A′ and B′ intersect) is virtual, and therefore lies inside the instrument, so that only a portion of the rays diverging from it can be received by an eye placed at the eye-piece.

For these reasons the Galilean telescope is best adapted for observation of terrestrial objects and where only a small magnifying-power is required. Opera glasses, field glasses, and some marine glasses are the principal forms of the instrument. These forms are usually *binocular*—that is, they consist of two telescopes with parallel axes, mounted so that both eyes may be conveniently used in looking at any object.

141. Pocket microscopes. We have already (Art. 130) discussed the use of a single convex lens on a magnifying glass. A lens of high power used in this way is called a **simple microscope**, and is most efficient if made plano-convex and used with its plane side towards the eye. The magnifying-power is inversely proportional to the focal length. In practice it is found that, as the focal length is decreased, distortion and chromatic defects creep in, and a single lens only acts well if its focal length be not less than one inch, so that for greater magnifying-powers recourse must be had to combination of lenses.

The simplest forms of pocket magnifiers are—

1. The **Coddington lens.** This was invented by Wollaston and is simply a sphere of glass (Fig. 157 A) in which a deep groove has been cut all the way round leaving only a small central aperture through which the rays may pass.

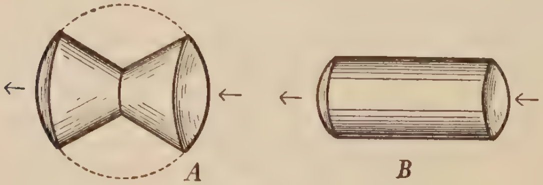

Fig. 157.

2. The **Stanhope lens.** This consists of a cylinder of glass (Fig. 157 B) whose ends are ground to spherical surfaces of unequal radii. The dimensions are so chosen that when a small object is placed on the end of the lesser curvature, an eye placed close to the other end sees a magnified and well-defined image.

142. The compound microscope. The compound microscope is in principle exactly similar to the astronomical telescope. The difference between the two instruments results from the adaptation of the object glass, or *objective* as it is called in the case of the microscope, to the purposes of the instrument. The objective of a microscope, O (Fig. 158), is essentially a convex lens of very short focal length. The small object to be viewed, A B, is placed close to O at a distance slightly greater than the focal length of that lens, and a real image of this object is formed at *a b* in front of the eye-piece O′. An eye looking through O′ sees, at A′ B′, a magnified virtual image of this already magnified real image, and by adjusting the position of O′ this virtual image can be seen at the nearest distance of distinct vision.

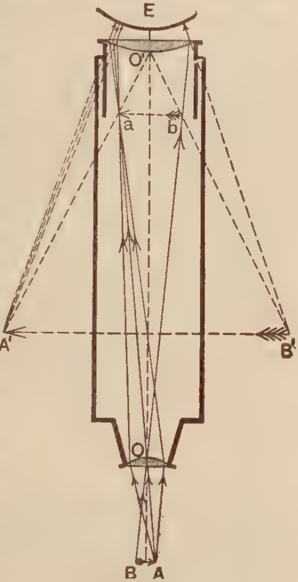

Fig. 158.

In actual instruments the objective is generally a complicated system of several lenses, constructed and arranged in order to diminish as far as possible the errors of aberration. Owing to the nearness of the object to the objective, the obliquity of the incident pencils is very great, and hence special precautions have to be taken to prevent excessive spherical aberration. The eye-piece and objective are in practice much more complicated than shown in Fig. 158.

Owing to the great magnification produced, the object requires to be very brilliantly illuminated, or the image at A′ B′ would be too faint to be of any use; hence most microscopes are provided with a reflecting mirror, mounted so that it can be easily placed so as to concentrate light on the object viewed. The magnifying-power is evidently determined, as in the case of the simple lens (Art. 130), by the ratio $\frac{A'B'}{AB}$, for both object and image are supposed to be seen at the nearest distance of distinct vision. But $\frac{A'B'}{AB} = \frac{A'B'}{a'b'} \cdot \frac{a'b'}{AB}$. Now $\frac{A'B'}{a'b'}$ is the magnification produced by the eye-piece, and is approximately equal to $\left(1 - \frac{D}{f}\right)$, where D is the nearest distance of distinct vision and f is the focal length of the eye-piece. Also $\frac{a'b'}{AB} = \frac{Oa'}{OA}$. Therefore the magnifying-power is approximately given by—

$$m = \left(1 - \frac{D}{f}\right) \frac{Oa'}{OA}.$$

But D, f, and O a are constants, for the same adjustment of the same eye-piece.

$$\therefore m \propto \frac{1}{OA}.$$

That is, the magnifying-power varies inversely as the focal length of the objective. For example, the magnification* produced by an objective of $\frac{1}{3}$ inch focal length is *approximately* three times that produced by an objective of 1 inch focal length.

Exp. 58. To determine the magnifying-power of a microscope. Adopt the method of Exp. 57 (2), using, however, two scales—one, a fine one, for observation through the microscope; the other, a coarser one, for observation with the naked eye. The magnifying-power is the true ratio of apparently equal lengths.

It is often found convenient to provide a microscope with a minutely divided scale in the eye-piece collar called a *micrometer scale*, for the purpose of measuring bodies of small dimensions. Before the absolute lengths can be got, the micrometer scale divisions must first be expressed in millimetres; and this is done by focussing the microscope upon a finely divided scale called a *stage micrometer*, graduated in millimetres and tenths, and noting how many divisions of the eye-piece scale are apparently equal to a millimetre. The object to be measured is then placed on the stage of the microscope, and its dimensions obtained in terms of the eye-piece scale divisions, and then by simple arithmetic calculated in millimetres.

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* It must be understood that throughout this chapter "magnification" means *linear* magnification.

There is no limit to vision. Any particle, however minute, can be seen as long as it can be suitably illuminated. If the dimensions of the particle are much less than half a wave-length of light it is only seen as a whole, *i.e.* its separate parts cannot be discriminated. The visibility of such particles is effected by focussing an intense beam of light upon them, and then viewing them through a suitably placed microscope, when they appear as bright points. The case is analogous to that of dust-motes, which, though as a rule invisible to the naked eye, are easily seen when a strong beam of sunlight passes through the air in which they are situated.

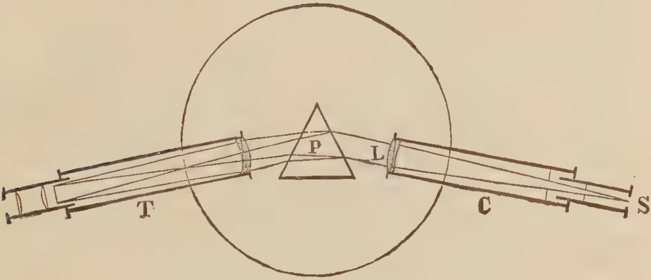

Fig. 159.

143. The spectroscope. This is an instrument constructed for the production and careful examination of pure spectra (Art. 95). It consists essentially of three parts—the *collimator*, the *prism*, and a *telescope*. The *collimator*, C (Fig. 159), is a tube, like a telescope tube with a *slit*, S, at one end, and a lens, L, at the other. The slit is an important part of the instrument; it consists of metal jaws with exactly parallel edges, and its width can be adjusted by means of an attached screw arrangement. The length of the collimator tube is such that when properly focussed the slit is at the principal focus of the lens.

The *prism*, P, is a short prism of glass of triangular cross section, similar to those referred to in Art. 65.

The *telescope*, T, is a small astronomical telescope similar to that described above in Art. 136. The arrangement of these three parts is shown in the figure; the telescope and collimator are attached to a central pillar and table, on which the prism is placed. The source of light whose spectrum is required is placed at S, so that the light from it falls directly on the slit of the collimator. The rays diverging from the slit are refracted through the lens L, and, emerging parallel, fall upon the prism P, where they again suffer refraction. Here each ray of the incident beam has the same angle of incidence, and rays of the same refrangibility will therefore be deviated to the same extent; hence, on emergence from the prism, the rays of each constituent of the dispersed beam will be parallel among themselves, though not to those of the other constituents. The position of the telescope is adjusted to receive this emergent beam; and, if the eye-piece is focussed for parallel light, a person looking into the telescope will see a magnified image of the pure spectrum which is formed in the focal plane of the object glass.

For purposes of measurement the instrument is often fitted with another tube, T′ (Fig. 160), carrying a scale at one end and a collimating lens at the other. This tube is placed with the lens facing the prism, and its position is so adjusted that an image of the scale, illuminated by some convenient source of light, is reflected from one face of the prism into the telescope. When the scale tube is properly focussed, the image of the scale will be seen in the telescope in coincidence with the spectrum, and the position of any line or band of the latter can be referred to its position on the scale.

By means of this scale the lines of any spectrum can be *mapped*—that is, their position, as given by the divisions of the scale, can be noted down. A map constructed in this way has, however, no absolute value, and would be different for different instruments, or for a different adjustment of the same instrument. We can, however, by a graphical construction, reduce it to an absolute scale of wave-lengths. By observing the position of a number of lines of *known* wave-length we can construct a curve, having as its abscissae the scale divisions and as ordinates the wave-lengths corresponding to given positions on the scale; then, by noting the position of any line on the scale, and measuring the ordinate of the curve corresponding to that position, we can determine approximately the wave-length of the line considered.

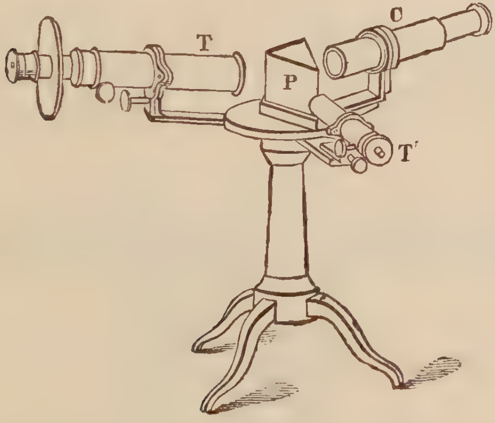

Fig. 160.

Very often it is necessary to compare two spectra. This is best done by arranging to obtain both spectra together in the same field. For this purpose a small right-angled total-reflection prism is fitted over the other half of the slit of the collimator, and a source of light, giving one of the required spectra, is placed on one side, so that the rays from it are totally reflected into the upper half of the slit. The other half is illuminated directly by the source giving the other spectrum, and thus both spectra are seen together, the one in the upper half of the field and the other in the lower half.

Fig. 160 shows a simple form of spectroscope.

In the best forms of the instrument there are two or more prisms instead of one. This is necessary in order to obtain greater dispersion; with one prism only a comparatively short spectrum can be obtained, and any peculiarities are not readily noticed. With a train of prisms so that the light passes successively through them the dispersion is increased by each prism, and a very long spectrum is obtained.

144. The spectrometer. This is a modification of the spectroscope adapted for accurate measurement.

Fig. 161 illustrates Wilson's* form for students. The collimator, C, is fixed to the base. The telescope, T, is mounted on a rotating piece which carries a circular scale graduated in degrees. The prism, P, is mounted on an adjustable table which can be levelled by three screws, and this in turn may be raised or lowered and afterwards clamped to the spindle of a circular plate which carries two verniers (180° apart) which slide alongside the circle attached

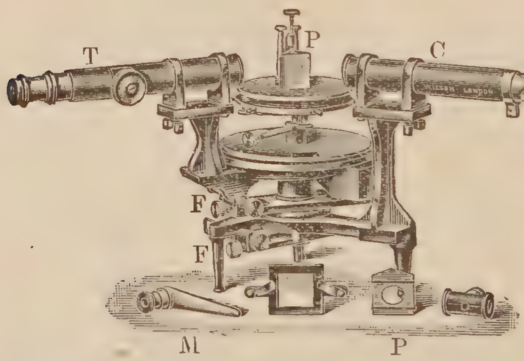

Fig. 161.

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* W. Wilson, 1, Belmont Street, Chalk Farm, N.W.

to the telescope. F, F are fine adjustments to be used after the prism and telescope are clamped in position. The verniers read to minutes of arc and for accurate work are read by a small magnifier, M.

Adjustments of the spectrometer. Before the spectrometer can be used it must be accurately adjusted. Any time spent over this process will be saved over and over again by the rapidity and accuracy with which the readings can be taken. The order in which the adjustments are performed is as follows:—

(1) The eye-piece is focussed on the cross wires.

(2) The telescope is focussed upon a distant object; the cross wires and eye-piece are in a small tube by themselves so that this adjustment does not disturb the previous one. The adjustment is correct when on moving the eye transversely across the eye-piece the image of the distant object remains at rest relative to the cross wires.

(3) The collimator and telescope are now brought into a straight line. The slit of the former is now illuminated and the adjustable tube on the collimator moved until the image of the slit as viewed through the telescope is distinctly focussed, the adjustment being tested as above.

145. Experiments with the spectrometer.

Exp. 59. The angles of a prism are easily measured by the spectrometer. Two methods are available; the agreement of the two sets of results is a test of the accuracy of the instrument and observer.

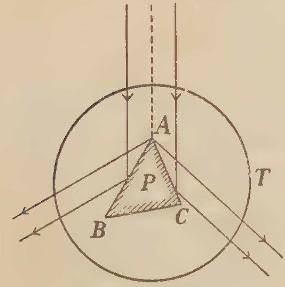

Fig. 162A.

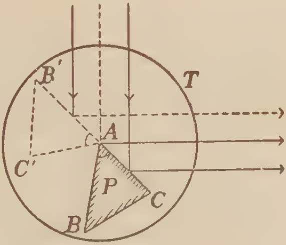

Fig. 162B.

(*a*) Mount the prism on the table, T, with the angle, A, which is to be measured pointing towards the collimator, and level T until the refracting edge of the prism is exactly vertical. Parallel light leaving the collimator is split into two parallel beams (Fig. 162A) by reflection at the prism faces A B, A C. Clamp the prism table in position. Sight the telescope upon each beam in turn, bring the images of the slit to the cross wire and read the verniers. It is obvious that the angle through which the telescope has been rotated is equal to twice the angle of the prism (cf. Art. 90).

(*b*) Mount the prism so that its refracting edge is just over the centre of the table. Rotate the telescope to a position about 90° from the collimator and clamp it. Now rotate the prism until the light reflected from the face A C, Fig. 162B, is sent down the telescope. Adjust until the image of the slit is on the cross wire, and read the verniers attached to the prism table. Next rotate the prism table until light reflected at the face A B is sent down the telescope and again read the verniers. The angle through which the prism has been turned is obviously equal to the supplement of the angle B A C. The other angles may be measured in like manner.

Exp. 60. The refracting angle of a prism being known, the determination of the refractive index of the material composing the prism is easily made. The slit is illuminated by monochromatic light, say by a sodium flame. First set the telescope to get a direct reading of the collimator slit, place the prism on the table in a suitable position, and locate the refracted image of the slit by the naked eye. Then bring the telescope around to view it and adjust to the position of minimum deviation as described in Art. 89. Take the reading of the telescope. The angle through which the telescope has been rotated from the first position is equal to D in the formula

$$\mu = \frac{\sin \frac{D+A}{2}}{\sin \frac{A}{2}}$$

from which μ can be readily calculated. If the refractive index of a liquid is required, the liquid may be placed in a hollowed glass prism (P′, Fig. 161), or in a prism whose sides are composed of three parallel-sided plates cemented together at the edges.

146. The direct vision spectroscope. This is a convenient form of spectroscope for the qualitative examination of flames and incandescent bodies. As explained in Art. 98 a crown and a flint glass prism may be combined to give dispersion without deviating the mean ray. By using several prisms with their edges alternately in opposite directions a very large dispersion may be obtained. The prisms are enclosed in a brass tube B, Fig. 163, provided at one end with an adjustable slit placed parallel to the refracting edges of the prism. Light leaving the slit S is rendered parallel by means of the lens L. It then falls on the prism combination, which may consist of three crown glass prisms united to two prisms of flint glass. The refracting angles and indices of refraction are so

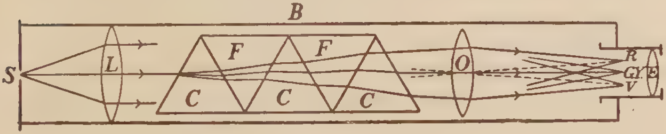

Fig. 163.

chosen that the brightest ray in the spectrum passes through without deviation, while the red and violet rays are deviated in opposite directions. Fig. 163 only shows the path of the rays arising from the central incident ray. The spectrum formed may be viewed directly or magnified by a short telescope, as in the figure. For their size these spectroscopes can be made very powerful.

147. Binocular vision. The stereoscope. If our eyes were fixed in their sockets our field of view, were we to keep the head fixed, would be very limited. The eyes, however, can be moved about 55° in every direction about their mean position, and distances are usually judged by the amount of convergence we have to impress upon the optic axes. It is extremely difficult to judge a distance accurately with one eye, as the reader will find if he tries to quickly place the point of his pen upon any small object on the table. Now to every point on one retina there is a corresponding point on the other, so that although two images of an object are formed by our eyes the brain is only cognisant of one.

When we look at a relatively near object of three dimensions, *i.e.* one having length, breadth, and thickness, the images formed on the retinas of the two eyes are not exactly alike, as the positions of the eyes are slightly different. This is rendered very apparent if we stand near and look at a clump of trees and quickly open and shut each eye in turn. The brain, however, blends these images into one, and the effort required for this gives us an idea of the solidity of the object. In the case of an ordinary picture the two images are almost exactly alike, hence the flatness which is nearly always very apparent; indeed, artists sometimes try to surmount this difficulty by exaggerating the perspective effects.

The stereoscope is an instrument invented by Wheatstone by which ordinary photographic pictures may be made to yield to the eye the appearance of depth. Two photographs A B, A′B′ (Fig. 164), of the same object are taken in two slightly different positions—the positions that a person's two eyes would be in if he were actually observing it from about the same position as that in which the camera is placed.

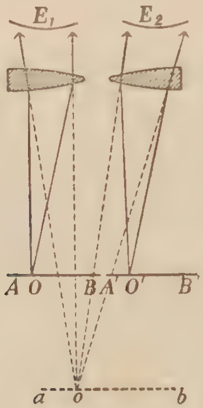

Fig. 164.

These are then correctly mounted on one card and then viewed through separate, very acute-angled prisms. These prisms are set with their angles inward; so that the rays from a point O in the right-hand picture, A B, are deviated outwards and enter the right eye, E_1, as if they were coming from a virtual image much to the left of O. Similarly the rays from the corresponding point O′ in the picture A′ B′ enter the left eye, E_2, as if they were coming from a virtual image to the right of O′. By varying the distance of the pictures A B and A′ B′ from the prisms, it is possible to make the virtual images of O and O′ coincide at a point *o*, say. At the same time, other virtual images will coincide, and thus, instead of two different pictures A B, A′ B′ being seen, only one—*a b*, a virtual image of these two—is perceived, and the impression produced on the mind of an observer is the same as if he were looking at the object itself, the front parts of the object appearing to stand out, and the back parts to sink back.

The surfaces of the prisms are usually curved convex, as indicated in the diagram, so that the images are magnified as well as superposed, thus making the detail much clearer. The best results are usually obtained when the two pictures are so mounted that the distance between corresponding points is nearly equal to the distance between a person's two eyes.

EXERCISES X.

1. An astronomical telescope is used to view an object placed at 20 yards distance, and the eye-piece is adjusted for nearest distinct vision of the image. Find the magnification, given that the focal length of the object glass is 2 feet, and that of the eye-piece 4 inches. What will be the magnifying-power of this instrument when adjusted for normal vision of a very distant object?

2. The images formed by the objective of a microscope are 8 inches from the objective. Find the magnifying-power of the instrument, given that the focal length of the objective is ½ inch, and that of the eye-piece 2 inches.

3. The focal length of the object glass of a telescope is 3 feet, and that of the eye-piece is 3 inches; draw a curve showing how the magnifying power varies with the distance of the object.

4. Find the focal length of a lens equivalent to a combination of two lenses, each of focal length f, and placed at a distance $2f$ apart.

5. Describe a simple form of microscope with two lenses, and trace pencils from different points of an object through it. If the rays emerge parallel to one another, what change must be made in the position of the lenses in order that the object may be clearly seen?

6. Describe the astronomical telescope; trace the course of a pencil of rays through it from any point of a distant object; and find the magnifying-power.

7. What is the magnifying-power of a telescope whose object glass is of 12 feet focal length, and its eye-piece of $\frac{1}{2}$ inch focal length?

8. In a Newtonian reflector whose speculum is of 10 feet focal length, what must be the focal length of the eye-piece to give a magnifying-power of 250?

9. Compare the light-grasping power of two mirrors whose diameters are 13 inches and 36 inches, and of the human eye when the pupil is $\frac{1}{4}$ inch in diameter.

10. A telescope is held with its object-glass end under the surface of the water of a pond; the water wets the outer surface of the glass, but does not come inside the telescope. The telescope is focussed so that objects at the bottom of the pond are clearly seen. Is the telescope now longer or shorter than when used for viewing objects at the same distance in air?

EXAMINATION QUESTIONS.

1. Describe an accurate method of verifying the laws of the reflection of light.

2. Describe the construction and use of Hadley's sextant.

3. How may a prism of glass be made so that no ray incident on one side will be refracted at the other side?

4. An arrow pointing towards the observer is seen by internal reflection in an isosceles right-angled prism. Explain the difference in, and give a sketch of, the images seen, according as the prism is a three-sided or a four-sided Wollaston prism.

5. When an object is to be photographed, an image of it is first obtained on a ground-glass screen by means of the lens of the camera. Describe the nature and position of this image, and explain its formation.

6. A simple lens is used as a magnifier. Sketch the relative positions of the object (an arrow) and its image. The same lens is used as in photography. Sketch the relative positions of the object and its image.

7. Describe the defects of the eye known as *short sight* and *long sight* respectively. Give careful diagrams showing how the defect is remedied in each case by the aid of a lens.

8. Explain, with the aid of diagrams, the use of spectacles in cases of *short sight* and *long sight* respectively.

Find the focal length of a concave lens which, placed just in front of a convex lens of focal length 0·9 inch, brings parallel rays to a focus one inch behind the convex lens.

9. A short-sighted person cannot see an object clearly at more than 6 inches distance from his eye. What kind of lens should he use, and what should be its focal length, if he wishes to read a book 12 inches away?

10. Assuming that the focal length of the lens of the eye can be varied with certain limits by muscular exertion, and that the lens is at a fixed distance of 15 mm. from the retina, find the focal length when an object (1) 30 cms., (2) 20 cms. away from the eye is seen distinctly. What will the focal length be when a very distant object is focussed on the retina?

11. Draw a diagram showing the passage of the rays through a telescope, with lenses of 1 inch and 9 inches focal length respectively, when used to view a distant source of light.

If the tubes containing the lenses can be drawn out until the lenses are 19 inches apart, show that an object 18 inches away can be focussed in the telescope.

12. Two convex lenses, one of 2 inches diameter and 6 inches focal length, the other of 1 inch diameter and 1 inch focal length, are arranged 7 inches apart so as to form a simple astronomical telescope. The instrument is then directed with the larger lens towards a star. Trace the course of rays from the star through it in a diagram drawn to scale. Show that the apparent brightness of the star is increased 36 times by the telescope.

13. What is the centre of a lens? Under what circumstances is the centre of a lens midway between its surfaces? Two equal lenses are placed side by side in the same plane, with their centres 3 inches apart. Two objects of the same size and shape, but of different colours, are placed behind the lenses at a distance of twice its focal length from each lens, and with their centres 6 inches apart, the line joining the centres of the objects being parallel to that joining the centres of the lenses. How will the images be situated, and what will be seen by an observer situated at a considerable distance in front of the lenses?

14. Describe Galileo's telescope (or an opera glass). State the connection between the relative positions of the two lenses and their focal lengths, and estimate the magnifying-power.

15. What do you understand by the chromatic dispersion of a lens? What is the best means of correcting it?

16. If you were observing a small luminous object by a telescope not corrected for chromatic aberration, what appearances would present themselves on sliding the eye-piece in and out?

17. You are given an astronomical telescope focussed for infinity and pointing at a daylit sky. You are also given a rule, a finely divided scale, and a pocket lens. How would you find the magnifying-power of the telescope, and how would you satisfy yourself that the result was not vitiated by stops in the telescope?

Give the theory of the method when the eye-piece is a single convex lens.

ANSWERS.

EXERCISES I. (*Pages* 13, 14)

9. 2·7 ft. **10.** 750 ft.
11. 844,000 miles. **12.** 100 cms. ; 20 cms.
13. ·0089 sq. cms. ; $A'B' = 13\frac{1}{3}$ cms.

EXERCISES II. (*Pages* 31, 32).

4. 10 ft. **5.** 80 cms. from the lesser light.
6. 4·5. **7.** 13·0. **8.** 11·2 ft.
9. 408 ins. **10.** (1) A dark spot, (2) a bright spot.
11. $L_B : L_C : L_D :: 8 : 3\sqrt{3} : 8$.
12. (*a*) Screen between the lamps, $2\frac{2}{3}$ ft. from 16-power lamp
(*b*) Screen outside lamp, 24 ft. beyond 16-power lamp.

EXAMINATION QUESTIONS (*Pages* 32, 33).

13. 900. **14.** $(2 + 3) : (3 + 4) : (4 + 2)$, *i.e.* $5 : 7 : 6$.
15. (1) Distance from B $33\frac{1}{3}$ cms., distance from A $66\frac{2}{3}$ cms.
(2) Distance from B 25 cms., distance from A 75 cms.
16. $\frac{1}{4}$. Halve the distance.

EXERCISES III. (*Pages* 57-59).

10. 2 ft. **12.** 12 ins., 24 ins., 36 ins., etc. ; 12 ins. ; 12 ins.
13. 60°. **14.** 3 ft. **15.** 100°. **18.** 8°36′.

EXERCISES IV. (*Pages* 85-87).

1. (1) Distance < 12; (2) distance < 12. **2.** 13·5 ins.
3. $\frac{I}{O} = -\frac{f}{u-f} = -\frac{f}{3f-f} = -\frac{1}{2}$, inverted. **4.** 12 ins.
5. ·025 in. ; 9 ins. behind. **6.** $3\frac{3}{7}$ ins. behind ; $\frac{3}{7}$.
7. $\frac{1}{2}$, virtual, $\frac{f}{2}$ behind. **9.** 2 : 1.

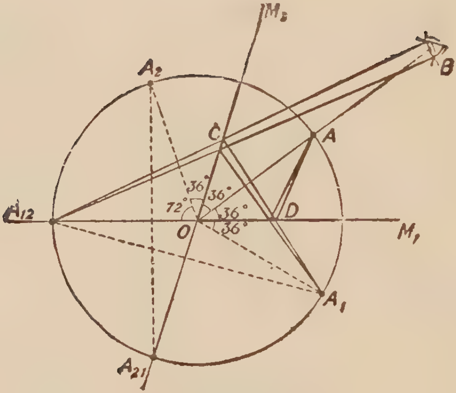

Fig. 165.

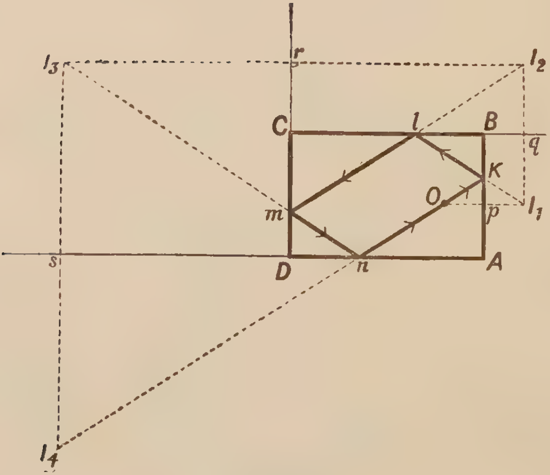

Fig. 166.

10. $2\frac{2}{3}$ ft. from the screen. **11.** $\frac{25}{26}$ ft.

12. ·51 ft. from the flame.

13. Real and one-third as large as the object ; 1 ft. from the mirror inverted. **14.** 6 ins.

15. 1 ft. from the mirror ; inverted ; three times as large ; at the centre of curvature.

16. The object is 3 ins. in front of the mirror, the image $1\frac{1}{2}$ ins. behind the mirror.

EXAMINATION QUESTIONS (*Pages* 87-89).

7. See Fig. 165.

12. Portions of the rippling surfaces are inclined at very different angles to the horizontal, hence many different rays from the moon have a chance of being reflected so as to enter an observer's eye.

13. See Fig. 166.

Yes, 4. See Fig. 167.

15. See Fig. 168. Image is $4\frac{1}{2}$ ft. from mirror, and half the size of the object.

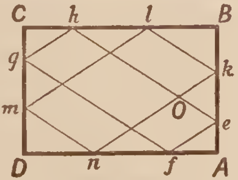

Fig. 167.

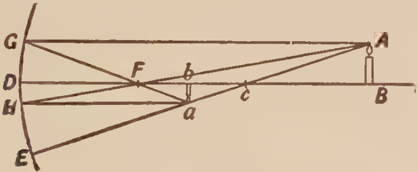

Fig. 168.

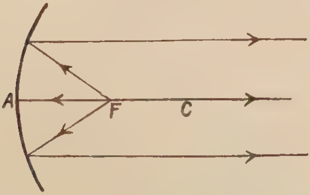

Fig. 169.

16. The arc is situated at the principal focus. See Fig. 169.

20. See Fig. 170.

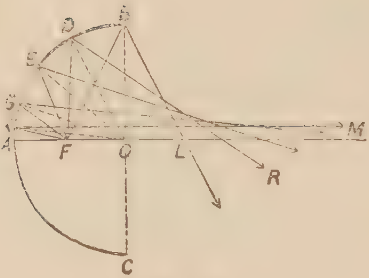

Fig. 170.

21. See Fig. 171.

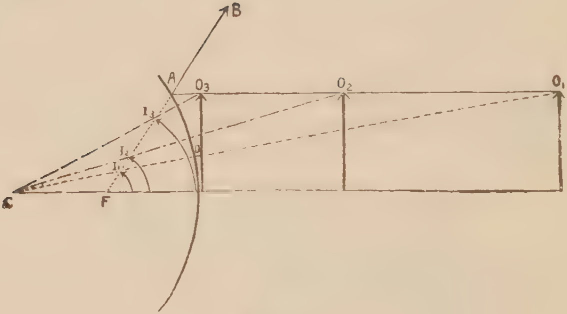

Fig. 171.

22. 2″ behind the mirror. See Fig. 172.

23. $r = \dfrac{2p\,(p+q)}{2p+q}$.

Exercises V. (*Pages* 117-119.)

1. $1\frac{7}{8}$ ins. nearer.

2. Relative refractive index = $\frac{5}{4}$. Angle of refraction = $34\frac{1}{2}$°.

3. $\frac{9}{7}$. **4.** 1·68. **6.** 1·15. **7.** 4·28 ins.

8. 225,000,000 metres per sec. **9.** 1·50. **10.** 1·491.

13. 30°. **19.** See last paragraph of Art. 60.

20. (1) The density of the air at any point is rapidly changing, hence also the refractive index. (2) Different parts of the solution have different refractive indices. **21.** Equal.

22. The point of incidence is 5 ft. from the bright point.

23. The images formed are 6 ins. and 42 ins. below the surface of the water.

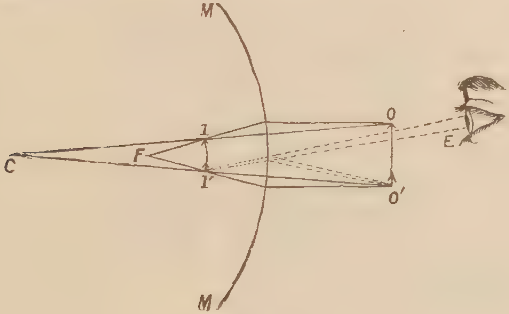

Fig. 172.

EXAMINATION QUESTIONS (*Pages* 119-122).

13. $\frac{40}{27}$.

21. Fish B appears at C, person A appears at D. **Fig. 173.**

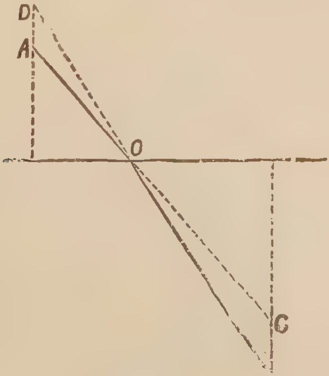

Fig. 173.

22. No.

23. See Exp. 24 and Fig. 174. Do not get the rays either nearly normal or nearly at grazing incidence.

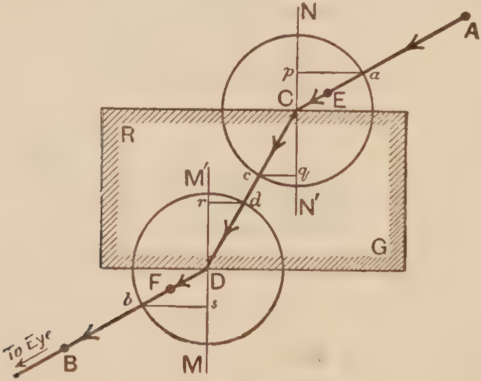

Fig. 174.

25. See Fig. 71. 1·09 cms.

26. See Fig. 175.

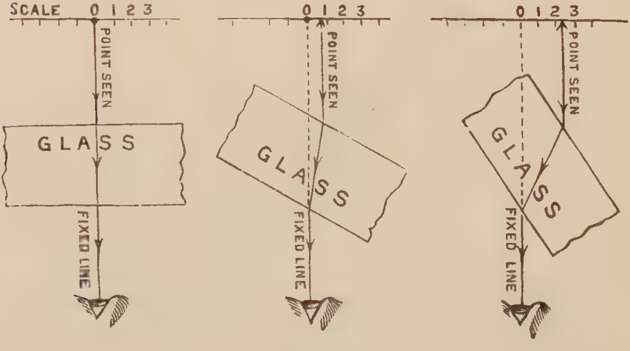

Fig. 175.

27. See Fig. 176. **28.** See Fig. 177. **29.** 1·73.

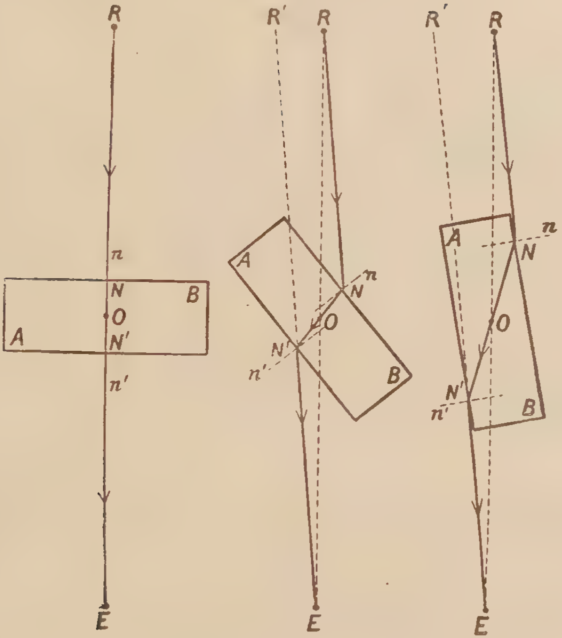

Fig. 176.

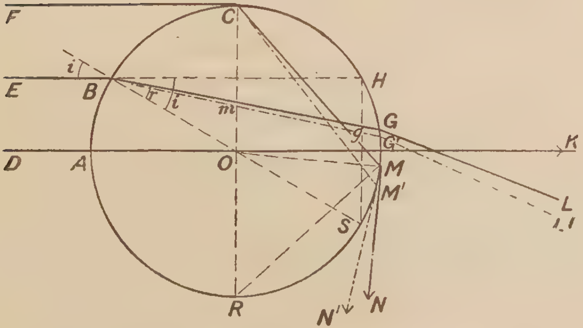

Fig. 177.
Full lines indicate red light, dotted lines indicate blue light.

EXERCISES VI. (*Pages* 158-160).

3. ·6°. **4.** 1·41.

6. $(3 \pm \sqrt{3})$ ft. from wall. **7.** Real ; 5 ins.

8. If image is real, $f = -8$ cms. ; if virtual, $f = -13\frac{1}{3}$ cms.

9. Virtual, $\frac{10}{7}$ object ; $4\frac{2}{7}$. **10.** 9 ins. **11.** 4 ins.

13. 10 ins. in front of plane mirror if light does not return through the lens, $2\frac{2}{11}$ ins. in front of lens if it does return through the lens. **14.** 15 cms. from mirror.

16. 25 : 1. **17.** 18 ins. (To obtain a *real* image with a *convex* lens.)

18. First image $8\frac{4}{7}$ ins. from first lens ; diameter $\frac{3}{7}$ in. Second image $15\frac{45}{61}$ ins. from second lens ; diameter $\frac{15}{61}$ ins.

19. 4 ins. **20.** 12 ins. from the lens on the side remote from the mirror.

22. 2 ft. from the lens on the same side as the object ; 6 ins. high.

23. 15 ins. ; the combination is concave. **24.** -12 ins.

25. 24 cms. **26.** 3.

EXERCISES VII. (*Page* 174).

1. -20 cms. **2.** $-10·02$ cms. **3.** 20·84 cms. **4.** 99·1 cms.

5. 100 cms. **6.** 10·7 cms., 1·53. **7.** 1·331. **8.** 1·628.

EXAMINATION QUESTIONS (*Pages* 175-177).

1. 1·73. Yes, as long as the critical angle is less than 30°.

For small angles the proof of Art. **66** may be modified thus :

$$\mu = \frac{\sin\phi}{\sin\phi'} = \frac{\phi}{\phi'}, \ \mu = \frac{\sin\psi}{\sin\psi'} = \frac{\psi}{\psi'},$$

$$\therefore \ \phi - \phi' = (\mu - 1)\,\phi' \text{ and } \psi - \psi' = (\mu - 1)\,\psi'.$$

Hence the total deviation which equals $\phi - \phi' + \psi - \psi'$

$$= (\mu - 1)(\phi' + \psi') = (\mu - 1)\,\mathrm{A}.$$

It is only possible to check it if μ has been calculated by other methods.

7. If the angle of the prism is greater than twice the critical angle no ray can cross the prism by a path similar to that of Fig. 76.

8. -5 ins.

13. See Fig. 178. Rays A and B come from top and bottom of sun respectively.

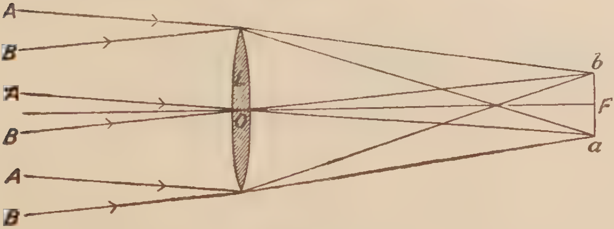

Fig. 178.

Diameter of image $= f \times \angle oab =$ ·63 in.

Brightness $\propto \left(\dfrac{\text{diameter}}{\text{focal length}}\right)^2$.

14. 2 ft. from lens and inverted. The eye should be placed so that the lens covers the object.

15. See Fig. 179.

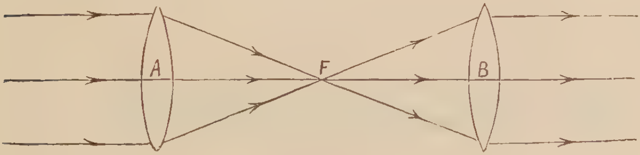

Fig. 179.

The emergent rays are parallel.

16. Convex, $6\frac{2}{13}$ ins., $126\frac{3}{4}$ ins. square. **17.** 36 ins.

18. 6 ins. in front of lens. **19.** $\frac{12}{13}$ in. from lens. Close to lens.

20. Let *mm* (Fig. 180) be a spherical surface, separating air from glass, C its centre of curvature, O a luminous point outside it. A ray O A is bent towards the normal C A at the point of incidence, and if O is sufficiently distant the refracted ray will meet the ray O C at a point I, which is the internal focus.

Call the angles A O C, A C O, A I O, α, γ, β as shown. Then

$$\angle \text{CAI} = \gamma - \beta \text{ and } \angle \text{DAO} = \alpha + \gamma.$$

Hence
$$\frac{\sin(\alpha+\gamma)}{\sin(\gamma-\beta)} = \mu.$$

If A is very near P, $\alpha+\gamma$ and $\gamma-\beta$ are small, and as the sines of small angles are nearly proportional to the angles,

$$\frac{\alpha+\gamma}{\gamma-\beta} = \mu \text{ nearly.}$$

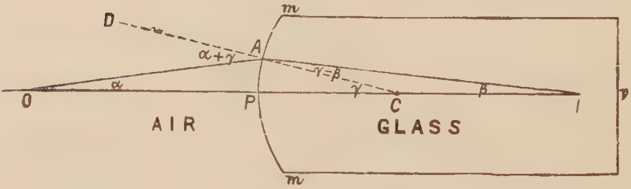

Fig. 180.

Now right-angled triangles on the same base, but of different heights, have the angles opposite the base nearly inversely proportional to the heights of the triangles if the angles are small.

Hence
$$\alpha : \beta : \gamma = \frac{1}{\mathrm{OP}} : \frac{1}{\mathrm{IP}} : \frac{1}{\mathrm{CP}}$$

Hence
$$\frac{\alpha+\gamma}{\gamma-\beta} = \frac{\frac{1}{\mathrm{OP}}+\frac{1}{\mathrm{CP}}}{\frac{1}{\mathrm{CP}}-\frac{1}{\mathrm{IP}}} \text{ and this } = \mu$$

Hence
$$\frac{\mu}{\mathrm{CP}} - \frac{\mu}{\mathrm{IP}} = \frac{1}{\mathrm{OP}} + \frac{1}{\mathrm{CP}}$$

or
$$\frac{\mu}{\mathrm{IP}} + \frac{1}{\mathrm{OP}} = \frac{\mu-1}{\mathrm{CP}}.$$
From this IP can be calculated.

22. See Fig. 181.

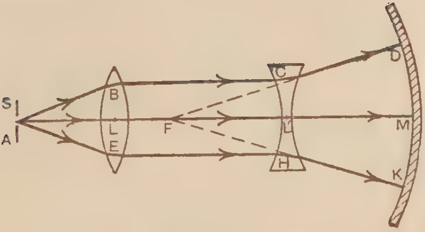

Fig. 181.

Radius of mirror − L'M = focal length of concave lens.

24. Focal length of mirror independent of colour. In case of lens, focal length for red light > focal length for blue light.

Exercises VIII. (*Page* 200).

4. Only the red end is seen. The rays from the rest of the spectrum are absorbed by the glass.

6. The red glass lets only red rays through, the blue glass only blue rays, hence when both used all the rays are stopped.

Examination Questions (*Pages* 200-202).

6. See Fig. 182. No, see Question 7, page 175.

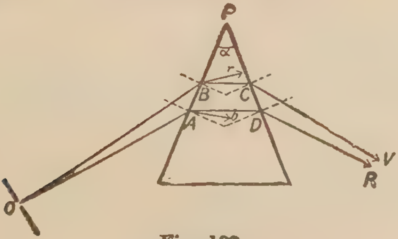

Fig. 182.

7. Diagram similar to Fig. 119.

10. See Fig. 183.

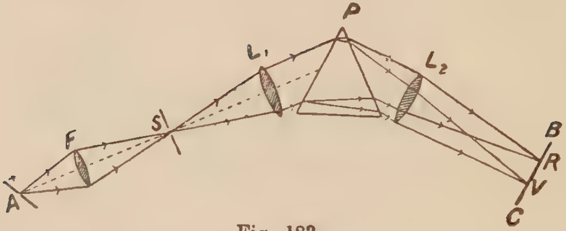

Fig. 183.

11. See Fig. 184.

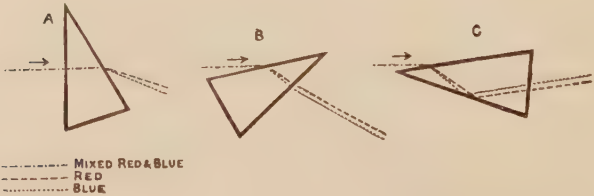

Fig. 184.

12. With blue light we get only a band of blue. With white light a spectrum. **17.** 2·56%.

18. No; decreased in fact because the red glass will absorb even a little red.

19. See Fig. 185.

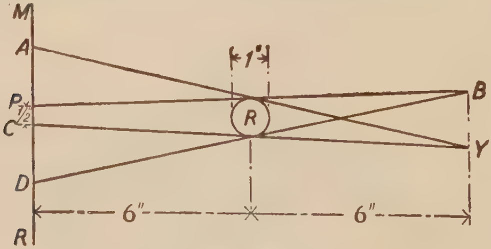

Fig. 185.

The colours on the screen are—going from side M to side R—white, blue, black, yellow, white.

EXAMINATION QUESTIONS (*Page* 215).

1. Least interval between successive eclipses is 40 hours minus 13·9 secs.; greatest interval is 40 hours plus 13·9 secs.

5. 1000 revolutions per sec.

6. 260 very nearly.

EXERCISES IX. (*Pages* 240, 241).

1. 8 ins. From 8 ins. to infinity.

3. 4.

4. $6\frac{2}{3}$ ins., $4\frac{1}{11}$ ins., 4 ins., concave lenses.

5. Convex; 20 ins. focal length.

6. Convex; $6\frac{2}{13}$ ins., $126\frac{3}{4}$ ins. square.

8. 2·4 ins.

9. An equilateral triangle.

10. Nearer. The rays converge to a point behind the retina.

11. Surrounding objects are greatly distorted.

EXERCISES X. (*Pages* 253, 254).

1. $8\frac{20}{29}$, 6.

2. 138.

4. $F = \dfrac{3f}{4}$.

7. 288.

8. $\frac{19}{25}$.

9. 2704 : 20740 : 1.

10. Outer surface of object glass plane, telescope same length; outer surface convex, telescope longer.

EXAMINATION QUESTIONS (*Pages* 262-264).

8. 9 ins.

9. Concave, 12 ins.

10. $-14\frac{2}{7}$ mm., $-13\frac{41}{43}$ mm., -15 mm.

11. See Fig. 186.

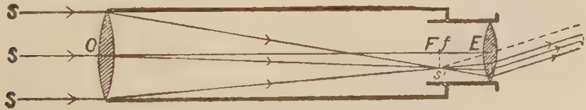

Fig. 186.

In second case the image of the object is still formed at *f*.

12. Magnification = 6. ∴ Areas are magnified 6^2 (or 36) times.

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$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}.$$

$$F = \boxed{\frac{f_1 f_2}{f_2 - f_1}}$$